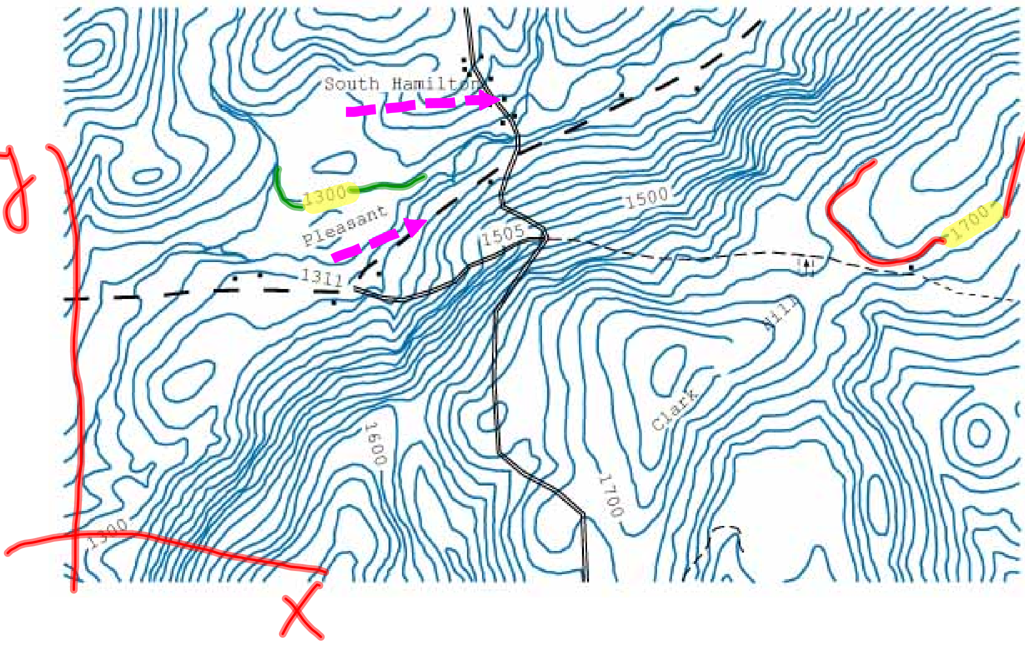
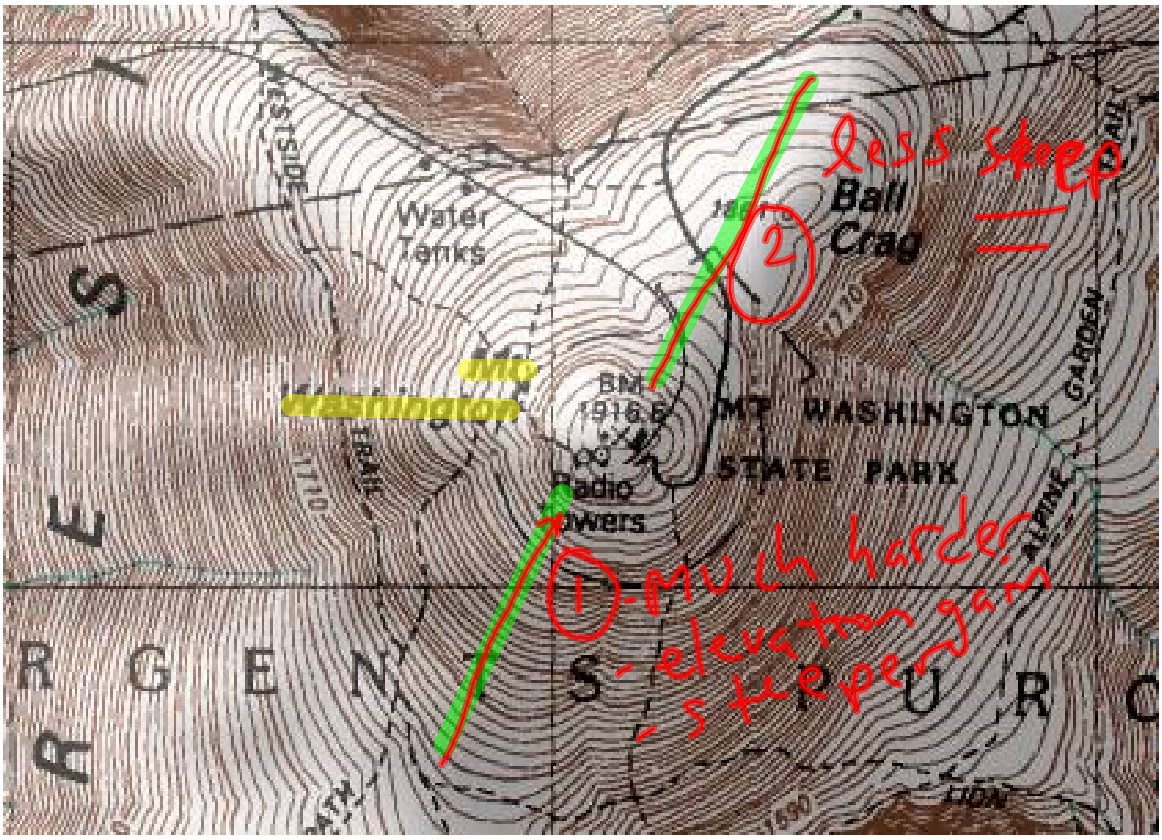
12.3 CONTOUR DIAGRAMS

3d shape onto 2d

Topographical Maps

z-value




Water
Tanks
Mt
Washington
BM
1916.6
Radio
Towers
Ball
Crag
MT WASHINGTON
STATE PARK
WESTSIDE
TRAIL
GARDEN
ALPINE
TRAIL
LION
PATH
1770
1710
1590
less steep
2
1 - much harder!
- elevation gain
- steeper

Contour lines (level curves or level sets)

Elevation

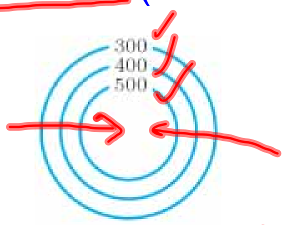


Figure 12.34 Mountain peak

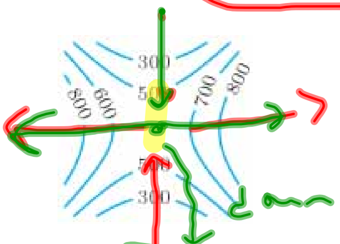
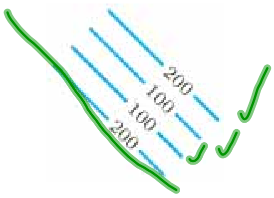


Figure 12.35 Pass between two mountains



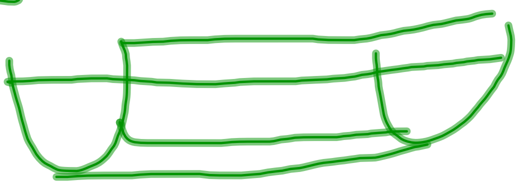


Figure 12.36 Long valley

Why can contour lines never intersect?

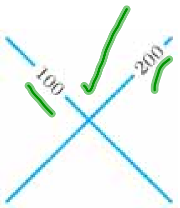
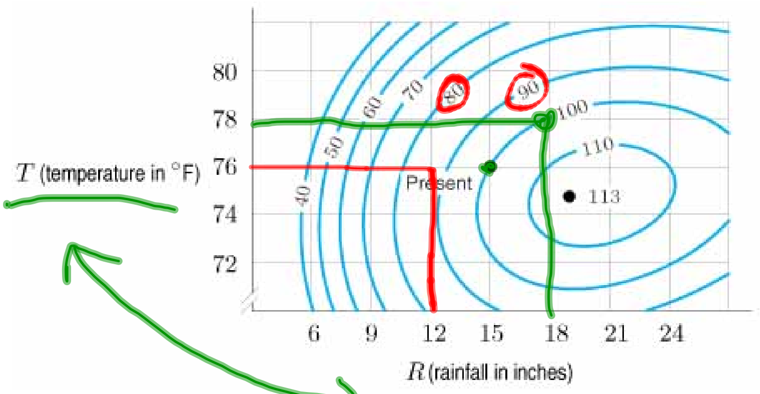


Figure 12.37 Impossible contour lines

Corn Production measured in percent of current yield

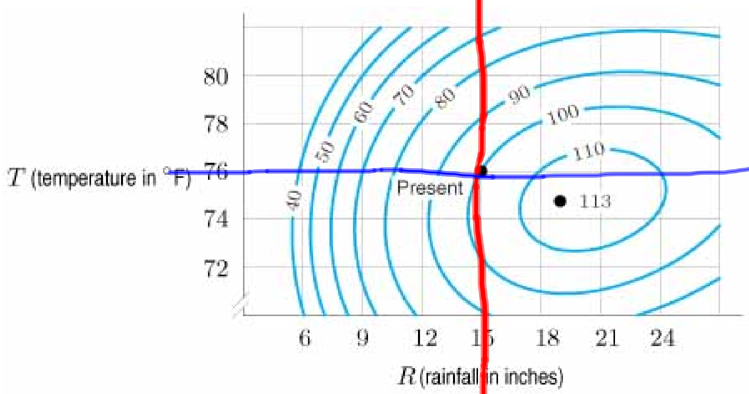


Use Figure 12.38 to estimate $f(18, 78)$ and $f(12, 76)$ and interpret in terms of corn production.

106%

80% < % < 90%

inc T, holding R, lose P



Hold T

inc R will increase P until R = 21

Use Figure 12.38 to describe in words the cross-sections with T and R constant through the point representing present conditions. Give a common sense explanation of your answer.

Contour Diagrams and Graphs

Contour diagrams and graphs are two different ways of representing a function of two variables. How do we go from one to the other? In the case of the topographical map, the contour diagram was created by joining all the points at the same height on the surface and dropping the curve into the xy-plane.

How do we go the other way? Suppose we wanted to plot the surface representing the corn production function $C = f(R, T)$ given by the contour diagram in Figure 12.38. Along each contour the function has a constant value; if we take each contour and lift it above the plane to a height equal to this value, we get the surface in Figure 12.39.

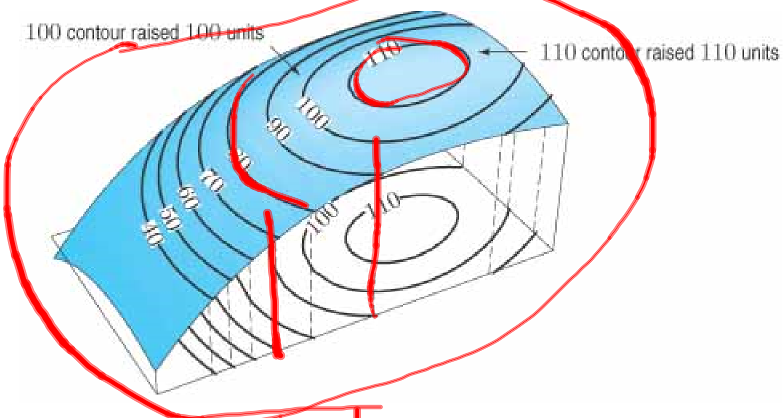


Figure 12.39 Getting the graph of the corn yield function from the contour diagram

LEVEL CURVES : z fixed

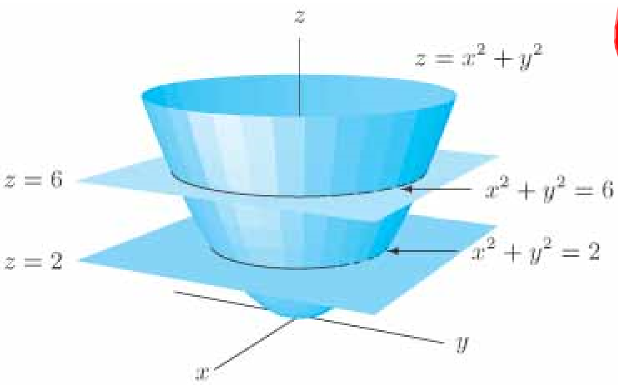
Contour lines, or level curves, are obtained from a surface by slicing it with horizontal planes.

Find equations for the contours of $f(x, y) = x^2 + y^2$ and draw a contour diagram for f. Relate the contour diagram to the graph of f.

$$\boxed{f(x, y) = x^2 + y^2} = c.$$

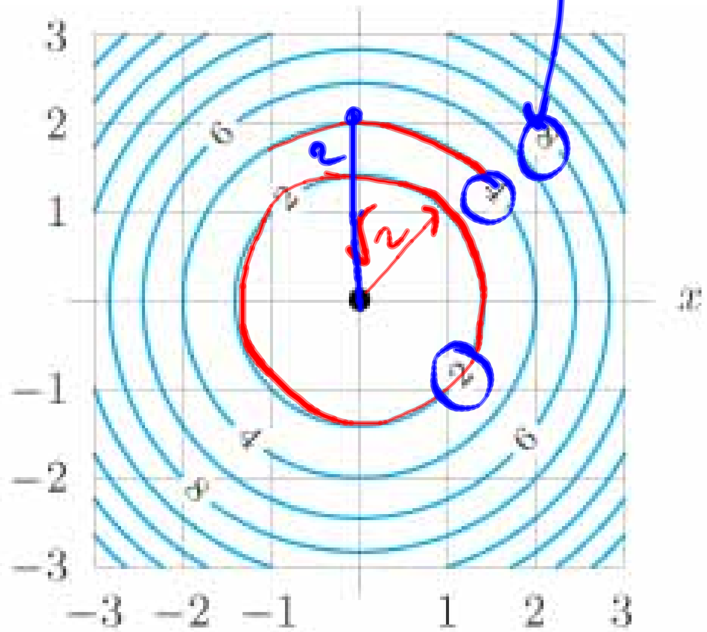
$c = 4$

$x^2 + y^2 = 4$



Draw a contour diagram of the following shape.

$z = x^2 + y^2 = c$



Draw contour lines.

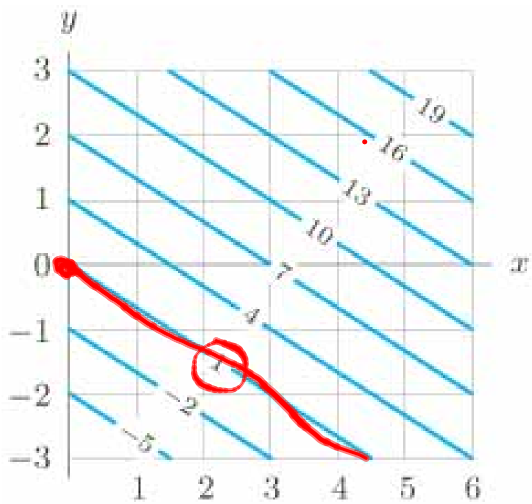
Fix z values

Draw a contour diagram for $f(x, y) = 2x + 3y + 1$.

Hmt

$z = 1$

$z=1$ $y = -\frac{2}{3}x$ $1 = 2x + 3y + 1$

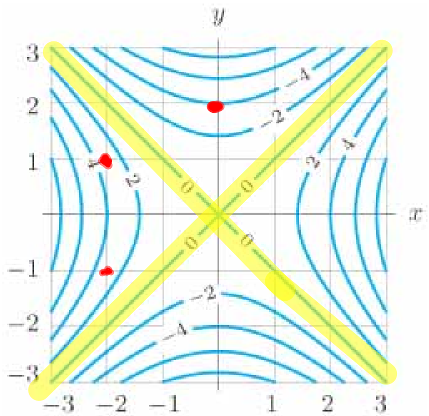


Contour Diagrams and Tables

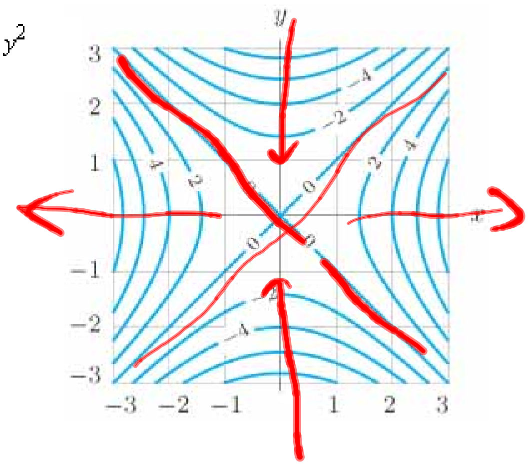
**Look at diagonal slopes!!
while we are not ready to completely understand why yet, the relationships are pretty intriguing!!!

Table 12.4 *Table of Values of $f(x, y) = x^2 - y^2$*

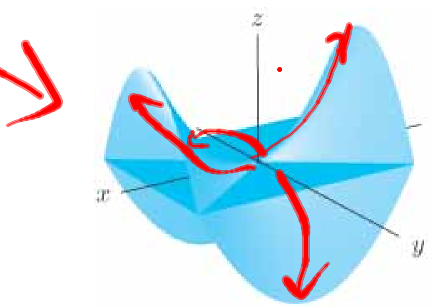
y \ x	-3	-2	-1	0	1	2	3
3	0	-5	-8	-9	-8	-5	0
2	5	0	-3	-4	-3	0	5
1	8	3	0	-1	0	3	8
0	9	4	1	0	1	4	9
-1	8	3	0	-1	0	3	8
-2	5	0	-3	-4	-3	0	5
-3	0	-5	-8	-9	-8	-5	0



Contour map of $f(x, y) = x^2 - y^2$

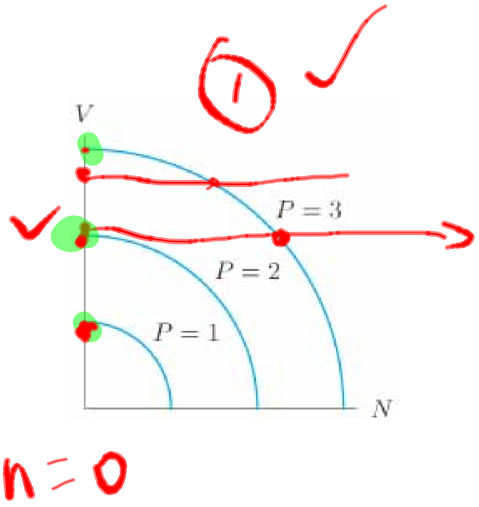


What would the figure look like?

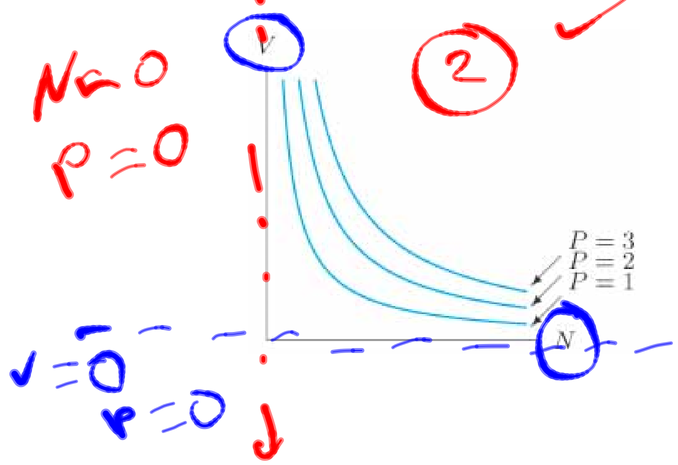


Using Contour Diagrams: The Cobb-Douglas Production Function

Let N= # of workers, V=value of equiptment, and P=production level. Which curve makes more sense



THINK ABOUT HOW TO GET TO THE NEXT P LEVEL, KEEPING ONE VARIABLE FIXED.



$P = cN^{\alpha} V^{\beta}$

Formula for a Production Function

Production functions are often approximated by formulas of the form

$$P = f(N, V) = cN^{\alpha}V^{\beta}$$

where P is the quantity produced and c, α, and β are positive constants, $0 < \alpha < 1$ and $0 < \beta < 1$.

Example 8

Show that the contours of the function $P = cN^{\alpha}V^{\beta}$ have approximately the shape of the contours in Figure 4.48.

last page

let $P = P_0$

$$P_0 = cN^{\alpha}V^{\beta}$$

$$V = \left(\frac{P_0}{c}\right)^{1/\beta} N^{-\alpha/\beta}$$

exponential decay

The contours are the curves where P is equal to a constant value, say P_0, that is, where

$$cN^{\alpha}V^{\beta} = P_0.$$

Solving for V we get

$$V = \left(\frac{P_0}{c}\right)^{1/\beta} N^{-\alpha/\beta}.$$

Thus, V is a power function of N with a negative exponent, so its graph has the shape shown in Figure 12.48.

The Cobb-Douglas Production Model

In 1928, Cobb and Douglas used a similar function to model the production of the entire US economy in the first quarter of this century. Using government estimates of P, the total yearly production between 1899 and 1922, of K, the total capital investment over the same period, and of L, the total labor force, they found that P was well approximated by the *Cobb-Douglas production function*

$$P = 1.01L^{0.75}K^{0.25}.$$

This function turned out to model the US economy surprisingly well, both for the period on which it was based, and for some time afterward.

P= total yearly production
K= total capital investment
L= total labor force