

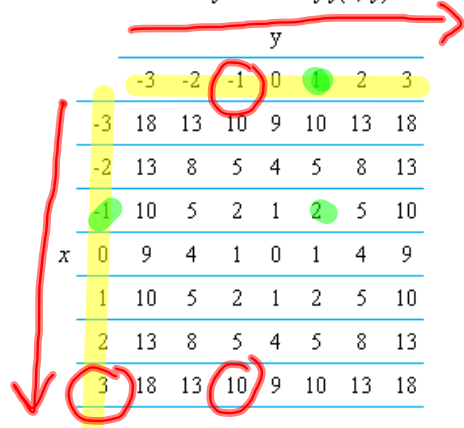
# New Material: 3d graphing

## 12.2 GRAPHS OF FUNCTIONS OF TWO VARIABLES

### Plotting the Graph of the Function $f(x, y) = x^2 + y^2$

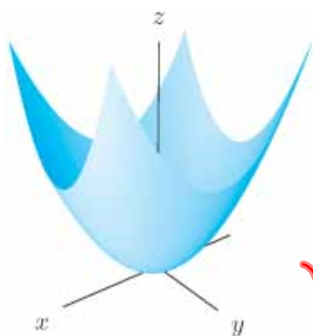
To sketch the graph of  $f$  we connect points as for a function of one variable. We first make a table of values of  $f$ , such as in Table 12.3.

Table 12.3 Table of Values of  $f(x, y) = x^2 + y^2$



	y						
	-3	-2	-1	0	1	2	3
-3	18	13	10	9	10	13	18
-2	13	8	5	4	5	8	13
-1	10	5	2	1	2	5	10
0	9	4	1	0	1	4	9
1	10	5	2	1	2	5	10
2	13	8	5	4	5	8	13
3	18	13	10	9	10	13	18

$$z > 0$$



3d  
parabola



Figure 12.13 Graph of  $f(x, y) = x^2 + y^2$  for  $-3 \leq x \leq 3$ ,  $-3 \leq y \leq 3$

use 3-d graphing... website on  
moodle

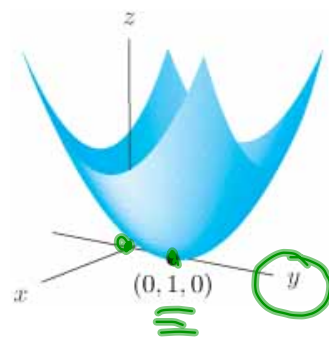
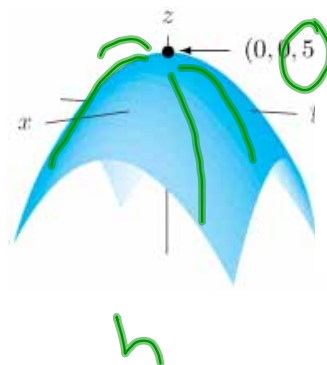
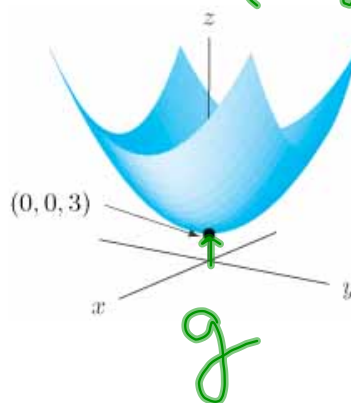
Find below some very helpful links.

- News forum
- Wolfram-Alpha (should be wolfram-awesome)
- MIT lessons
- Great 3-d Grapher and contour diagram
- Helpful applets
- More helpful applets
- Sign up for Khan Academy.
- gradients and level curve apps
- Polar Grapher
- Homework Assignment Due Dates

# TRANSFORMATIONS ☒

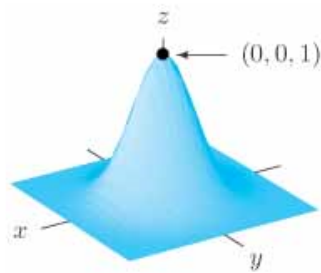
Let  $f(x, y) = x^2 + y^2$ . Describe in words the graphs of the following functions:

- (a)  $g(x, y) = \underline{x^2 + y^2} + 3$ , shift + up +3 z direction  
 (b)  $h(x, y) = 5 - \underline{x^2 - y^2}$ ,  $5 - (x^2 + y^2)$ , 5 → up z  
 (c)  $k(x, y) = x^2 + \underline{(y - 1)^2}$ , "—" → flip parabola  
 shift + (+1) y direction



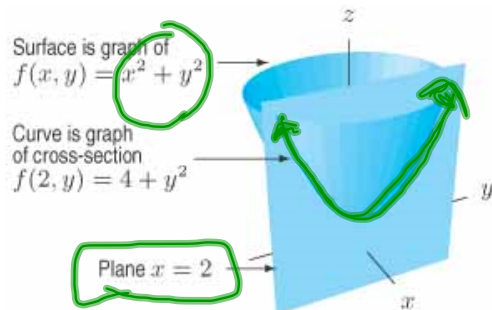
Describe the graph of  $G(x, y) = e^{-(x^2+y^2)}$ . What symmetry does it have?

$\rightarrow$  -circle  
 $\rightarrow x^2 + y^2 > 0$   
 $e^{-\#}$   
 $0 < G < 1$   $\rightarrow x=y=0$

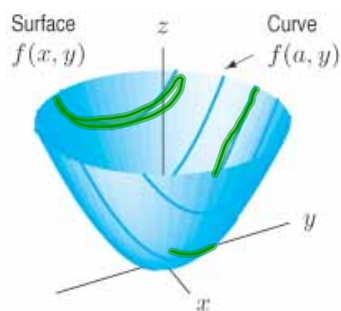


## Cross-Sections and the Graph of a Function

For a function  $f(x, y)$ , the function we get by holding  $x$  fixed and letting  $y$  vary is called a **cross-section** of  $f$  with  $x$  fixed. The graph of the cross-section of  $f(x, y)$  with  $x = c$  is the curve, or cross-section, we get by intersecting the graph of  $f$  with the plane  $x = c$ . We define a cross-section of  $f$  with  $y$  fixed similarly.

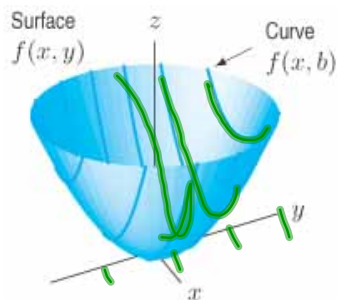


$$f(x, y)$$
$$f(2, y) = 4 + y^2$$
$$z = 4 + y^2$$



✓✓  $f_{-x}$   
x

**Figure 12.19** The curves  $z = f(a, y)$  with  $a$  constant: cross-sections with  $x$  fixed



$z = x^2 + \underline{b^2}$

**Figure 12.20** The curves  $z = f(x, b)$  with  $b$  constant: cross-sections with  $y$  fixed

Describe the cross-sections of the function  $g(x, y) = x^2 - y^2$  with  $y$  fixed and then with  $x$  fixed. Use these cross-sections to describe the shape of the graph of  $g$ .

Fix  
X


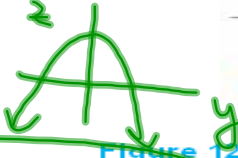
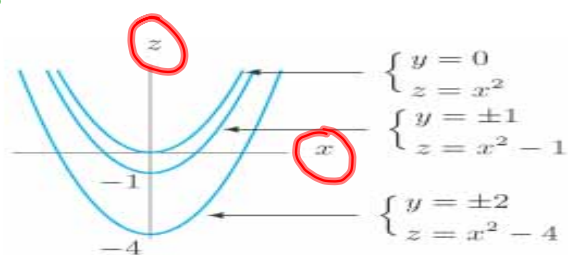
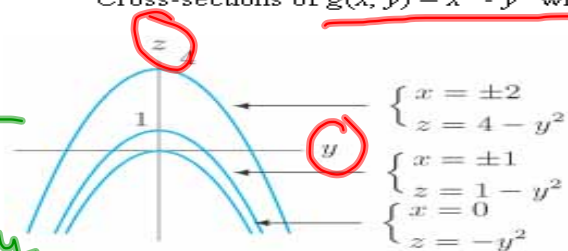
$x$	$g(x, y)$	
0	$-y^2$	
$\pm 2$	$4 - y^2$	

Figure 12.21

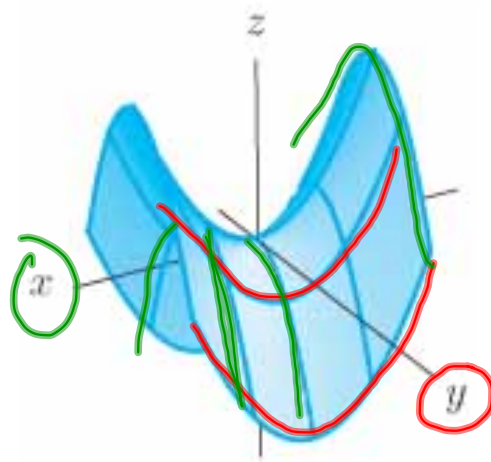


Cross-sections of  $g(x, y) = x^2 - y^2$  with  $y$  fixed



Cross-sections of  $g(x, y) = x^2 - y^2$  with  $x$  fixed





**Figure 12.23** Graph of  $g(x, y) = x^2 - y^2$  showing cross sections

saddle

## When One Variable is Missing: Cylinders

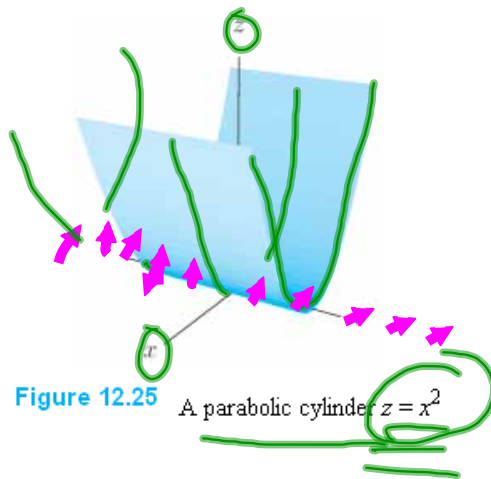
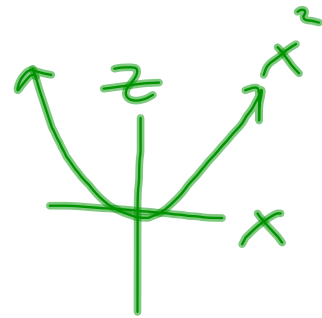


Figure 12.25

A parabolic cylinder  $z = x^2$

$y$  can be anything

Graph the equation  $x^2 + y^2 = 1$  in 3-space.

circle;  $r=1$   
 $z$  can be anything

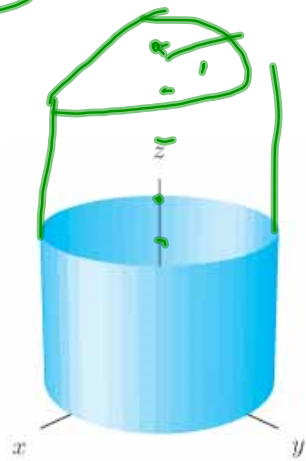
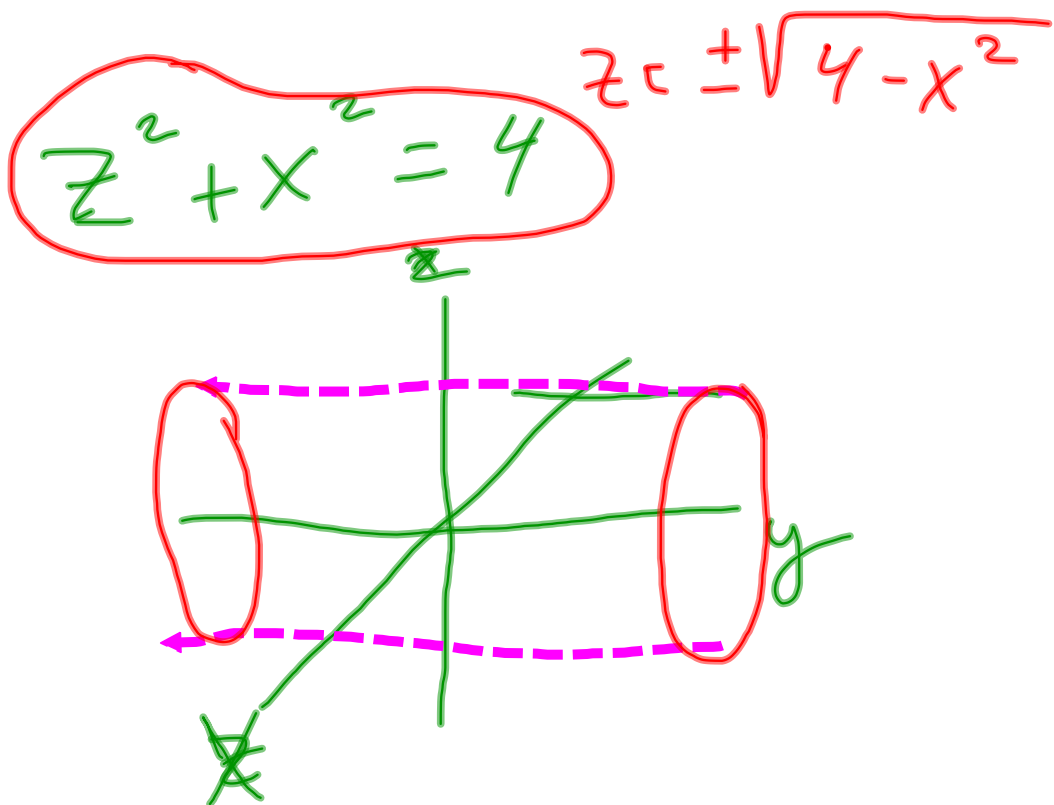


Figure 12.26 Circular cylinder  $x^2 + y^2 = 1$



## Graphing 3d

- cross section
- missing variables
- comp software ✓