

## MULTIVARIABLE

### 12.1 FUNCTIONS OF TWO VARIABLES

dependent

$$\boxed{m} = f(L, r)$$

rate ☒

LOAN ☒

You pay monthly car payments.

What do the payments depend on?

Loan amount and interest rate.

What are the variables? What is the independent variable? Dependent?

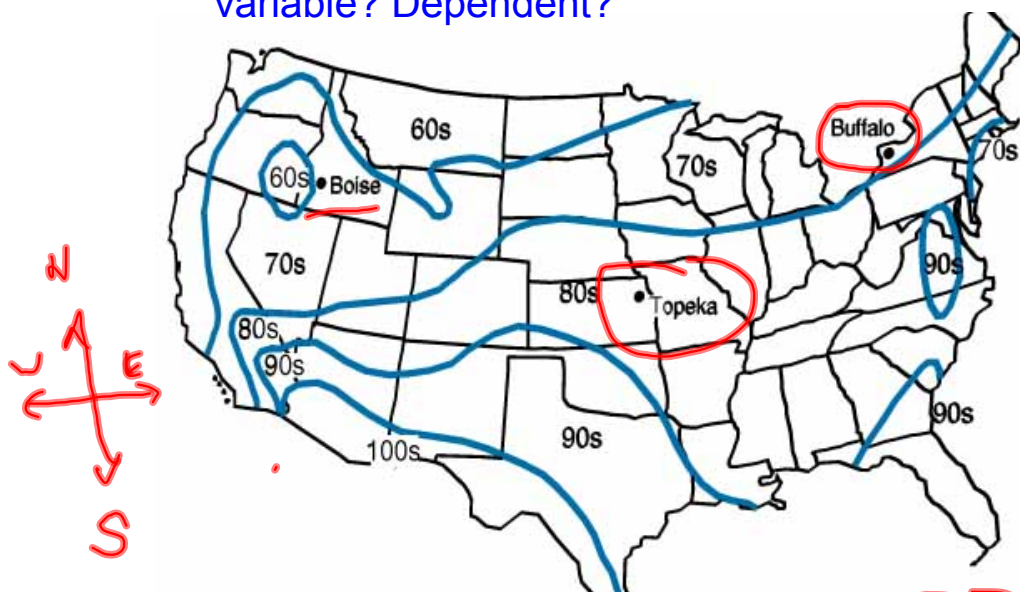


Figure 12.1 Weather map showing predicted high temperatures,  $T$ , on a summer day

What can you infer about the temperatures in the cities given?

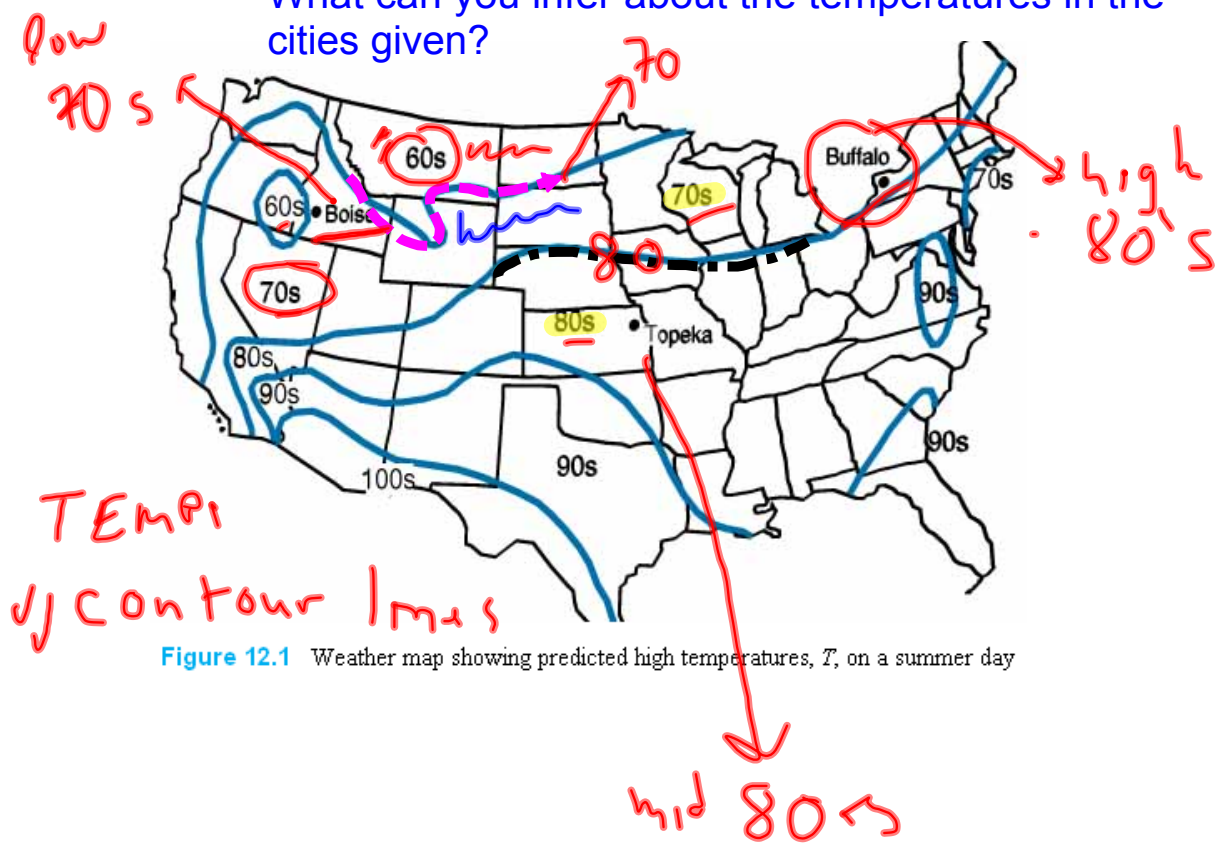
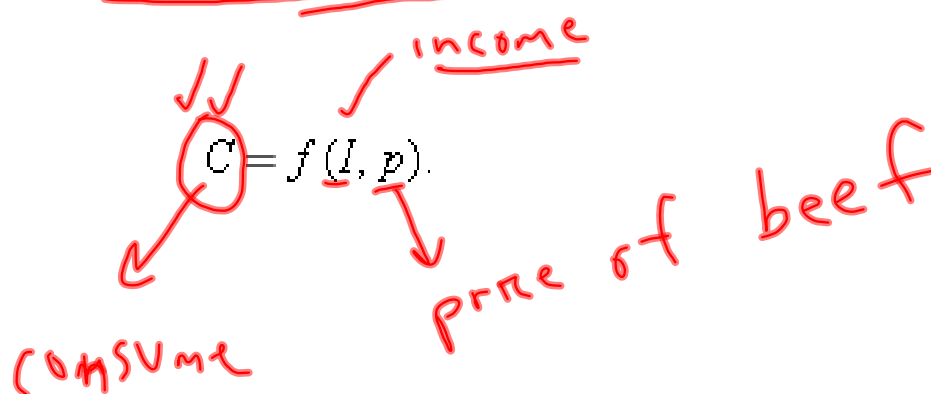


Figure 12.1 Weather map showing predicted high temperatures,  $T$ , on a summer day

## Numerical Example: Beef Consumption

What factors would influence how much beef the average family consumes?



**Table 12.1** *Quantity of Beef Bought (Pounds/Household/Week)*

	Price of beef, (\$/lb)				
	3.00	3.50	4.00	4.50	
20	2.65	2.59	2.51	2.43	
40	4.14	4.05	3.94	3.88	
Household income per year, $I$ (1000)	60	5.11	5.00	4.97	4.84
	80	5.35	5.29	5.19	5.07
	100	5.79	5.77	5.60	5.53

How much data is conveyed in each cell?

3 pieces of info

1)  $C$

2)  $I$

3)  $P$

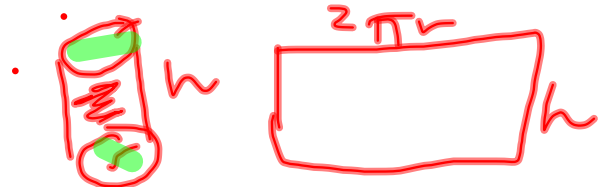
$$y = y_0 e^{rt}$$

Give a formula for the function  $M = f(B, t)$  where  $M$  is the amount of money in a bank account  $t$  years after an initial investment of  $B$  dollars, if interest is accrued at a rate of 5% per year compounded annually.

$$M(B, t) = M = B e^{0.05t}$$

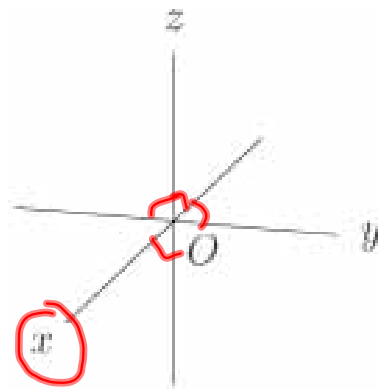
A cylinder with closed ends has radius  $r$  and height  $h$ . If its volume is  $V$  and its surface area is  $A$ , find formulas for the functions  $V = f(r, h)$  and  $A = g(r, h)$ .

$$V = \pi r^2 h$$



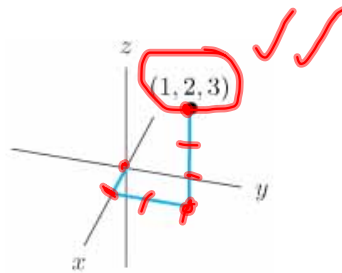
$$A = 2\pi r \cdot h + 2(2\pi r^2)$$

## A Tour of 3-Space



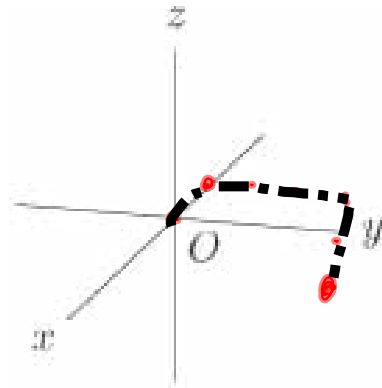
comes  
out of  
page

$(x, y, z)$   
Plot the point  $(1, 2, 3)$

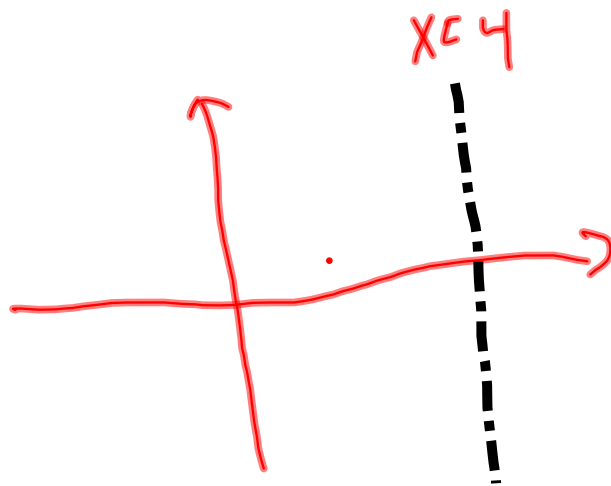




More Practice Plotting :  $(-1, \underline{3}, \underline{-2})$

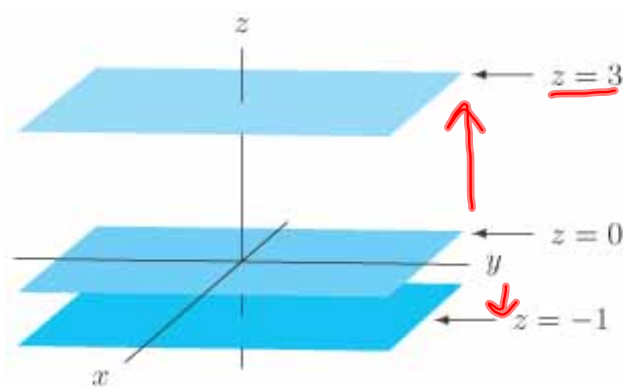


Preview: In 2-d space, what does the  
equation  $x=4$  look like?



$x, y$  can be anything

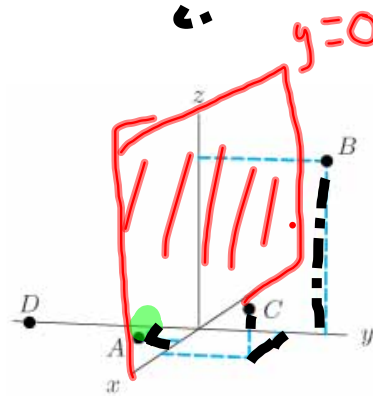
What do the graphs of the equations  $z = 0$ ,  $z = 3$ , and  $z = -1$  look like?



planes

→  $xy$  plane

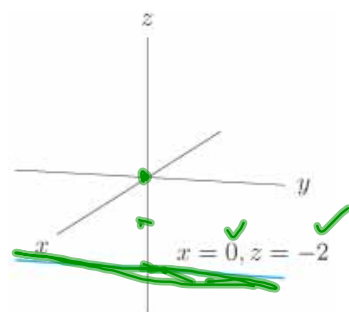
Which of the points  $A = (1, -1, 0)$ ,  $B = (0, 3, 4)$ ,  $C = (2, 2, 1)$ , and  $D = (0, -4, 0)$  lies closest to the  $xz$ -plane? Which point lies on the  $y$ -axis?



$x, z$   
can be  
anything.  
 $y=0$

You are 2 units below the  $xy$ -plane and in the  $yz$ -plane. What are your coordinates?

$$z = -2$$



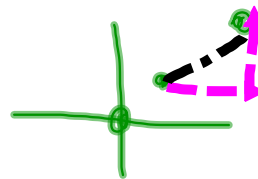
$$x = 0$$


$$z = -2$$

$$y = ?$$

Intersection of two planes:  
line

## Distance between Two Points



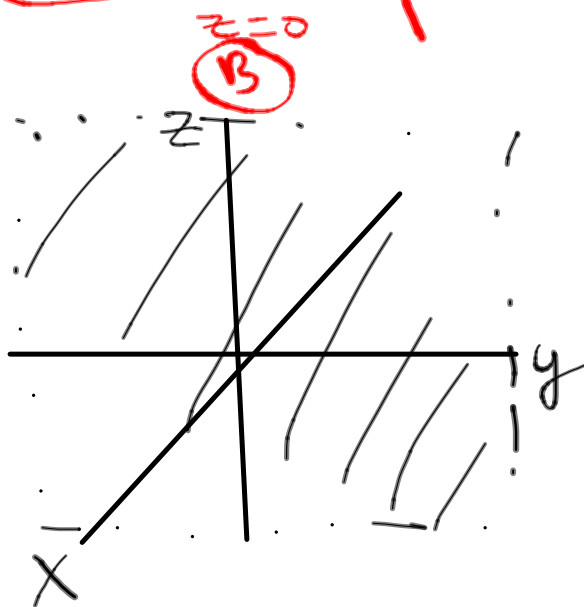

$$\text{Distance} = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$

Find the distance between  $(1, 2, 1)$  and  $(-3, 1, 2)$ .

$$\begin{aligned} & \sqrt{4^2 + 1^2 + (-1)^2} \\ &= \sqrt{18} \quad = 3\sqrt{2} \end{aligned}$$

Restart ✓

2. Which of the points  $A = (1.3, -2.7, 0)$ ,  $B = (0.9, 0, 3.2)$ ,  $C = (2.5, 0.1, -0.3)$  is closest to the  $yz$ -plane? Which one lies on the  $xz$ -plane? Which one is farthest from the  $xy$ -plane?

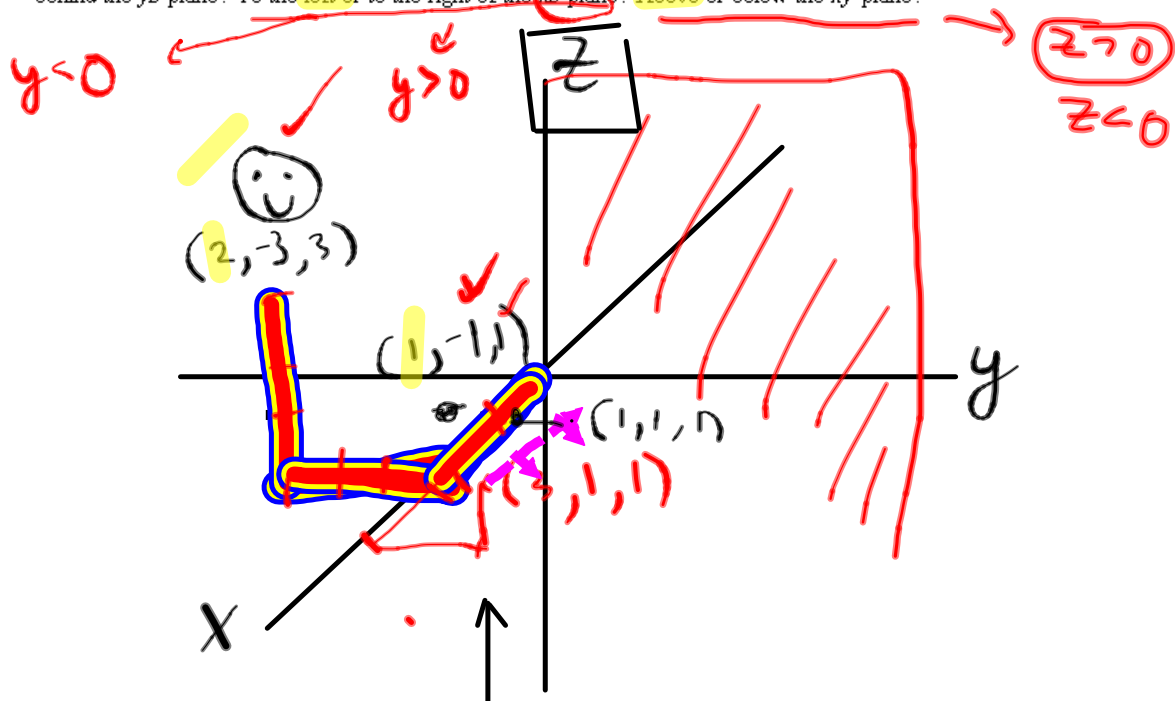


$x=0$

$y=0$

$y, z$   
can be  
anything

5. You are at the point  $(-2, 1)$ , standing upright and facing the  $yz$ -plane. You walk 2 units forward, turn left, and walk another 2 units. What is your final position? From the point of view of an observer looking at the coordinate system in Figure 12.2, are you in front of behind the  $yz$ -plane? To the left or to the right of the  $xz$ -plane? Above or below the  $xy$ -plane?



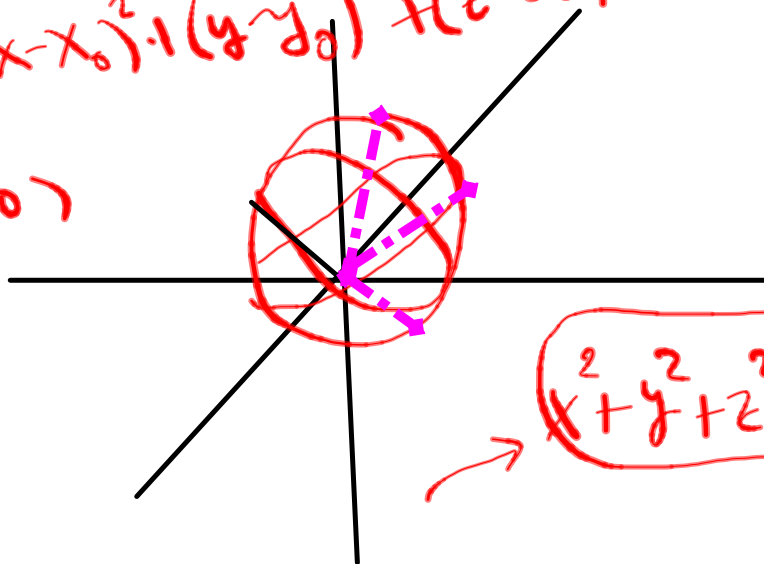


10. Find the equation of the sphere of radius 5 centered at the origin.

all points are a distance of 5  
from origin

$$d = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

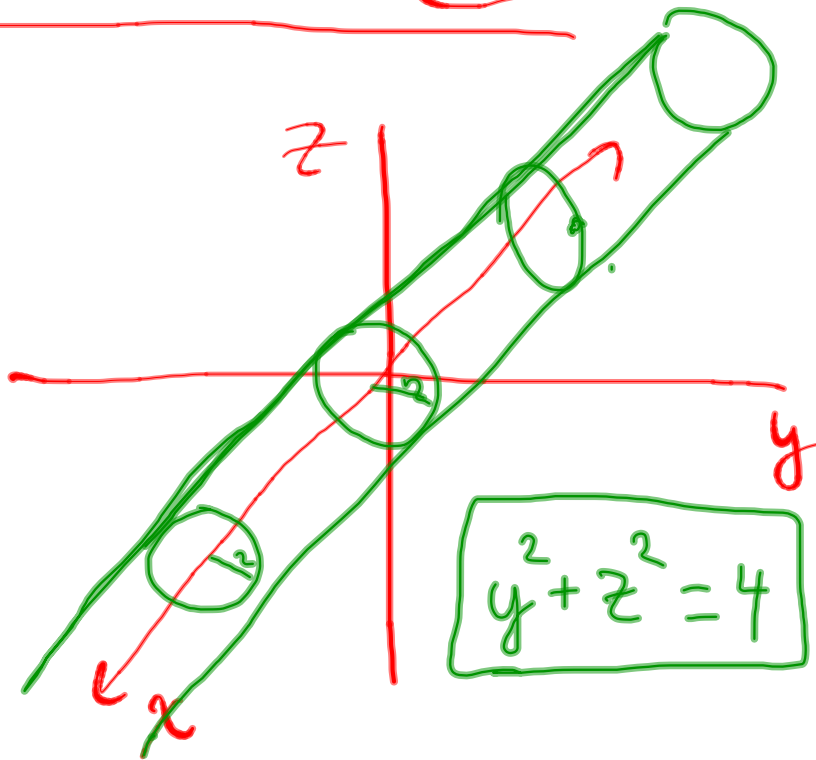
$(0,0,0)$



$$x^2 + y^2 + z^2 = 25$$

$$5 = \sqrt{x^2 + y^2 + z^2}$$

28. Describe the set of points whose distance from the  $x$ -axis is 2.



3-d

- distance

- proximity of  
pts to a plane