5.1: increasing and decreasing functions

- If $f' > 0$ on an interval, then f is increasing on the interval.
  If $f' < 0$ on an interval, then f is decreasing on the interval.
- If $f'' > 0$ on an interval, then f' is increasing and f is concave up on the interval.
  If $f'' < 0$ on an interval, then f' is decreasing and f is concave down on the interval.

f' $+,-$ $-,+$

Find the local max and min of the following function.

$f(x) = x^3 + 3x^2 - 24x + 18?$

$f'(x) = 3x^2 - 6x - 24$

$= 3(x^2 - 2x - 8)$

$f'(x) = 3(x-4)(x+2) = 0$

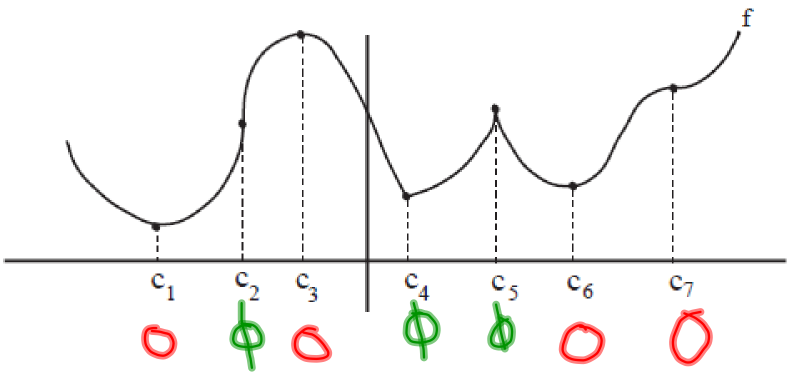
$x = -2, 4$

x	$(-\infty, -2)$	-2	$(-2, 4)$	4	$(4, \infty)$
$f'(x)$	$+$	0	$-$	0	$+$

$x = -2$: max

min $x = 4$

A number c in the interior of the domain of a function f is called a **critical number of f** if either $f'(c)=0$ or $f'(c)$ is not defined.
The point $(c, f(c))$ is called a **critical point** of the graph of f.



Find the critical numbers for $f(x) = x^5 - 5x^4 + 5x^3 + 20$

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

$$= 5x^2(x^2 - 4x + 3)$$

$$= 5x^2(x-3)(x-1)$$

$$c = 0, 1, 3$$

Find the critical numbers for $g(x) = \dfrac{e^x}{x-2}$

$$g'(x) = \frac{(x-2)e^x - e^x(1)}{(x-2)^2}$$

$$= \frac{e^x(x-2-1)}{(x-2)^2}$$

$g'(x) = 0, \phi$ $\qquad x = 3, 2$

Find the critical numbers for $h(x) = x^{3/5}(4 - x)$

$h(x) = 4x^{3/5} - x^{8/5}$

$h'(x) = 4 \cdot \frac{3}{5} x^{-2/5} - \frac{8}{5} x^{3/5}$

$= \frac{12}{5x^{2/5}} - \frac{8x^{3/5}}{5} \cdot \frac{x^{2/5}}{x^{2/5}}$

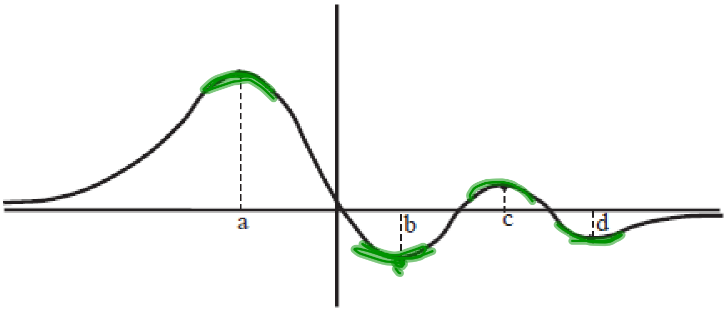
$= \frac{12}{5x^{2/5}} - \frac{8x}{5x^{2/5}}$

$= \frac{12 - 8x}{5x^{2/5}} = \frac{4(3 - 2x)}{5x^{2/5}}$

$h'(x) = 0, \varnothing$

$\boxed{x = \frac{3}{2}, 0}$

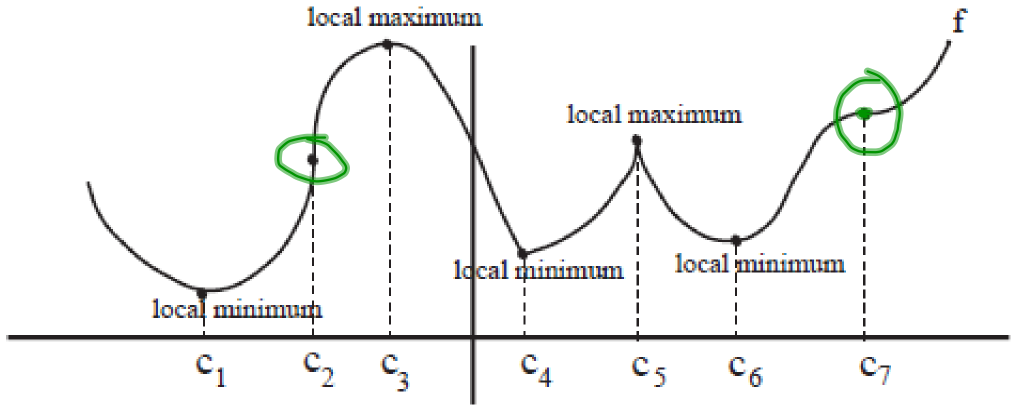
local max and min



Suppose that c is a critical number of a function f. Then

- f has a **local minimum** at $x = c$ if, for all x near c, $f(c) \le f(x)$;
- f has a **local maximum** at $x = c$ if, for all x near c, $f(c) \ge f(x)$.

critical points versus local max and min

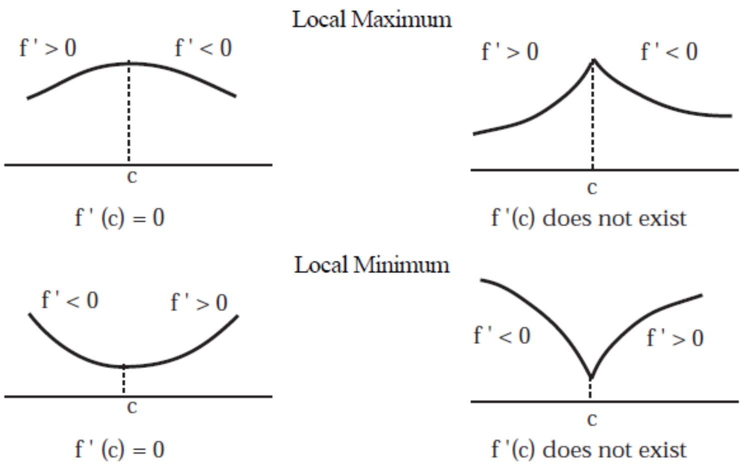


where is a critical pt w/o a max or min?

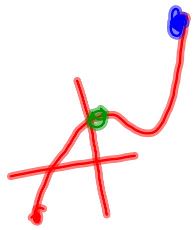
The First Derivative Test

If c is a critical number and if f' changes sign at $x = c$, then

- f has a local minimum at $x = c$ if f' is negative to the left of c and positive to the right of c;
- f has a local maximum at c if f' is positive to the left of c and negative to the right of c.



Global Max, Local Max and Extreme Value Theorem



A function f has

- a **global maximum value** $f(c)$ at the input c if $f(x) \le f(c)$ for every x in the domain of f;
- a **global minimum value** $f(c)$ at the input c if $f(x) \ge f(c)$ for every x in the domain of f.

will occur @ critical point or @ endpoint

The Extreme Value Theorem

If a function f is continuous on a closed interval $[a, b]$, then f has a global maximum and a global minimum value on $[a, b]$.

Step 1. Find the critical numbers that lie in the interval (a, b). ✓

Step 2. Calculate $f(c)$ for each critical number c. ✓

Step 3. Calculate endpoint values $f(a)$ and $f(b)$. ✓

Step 4. Compare outputs at all critical numbers and endpoints to identify the global maximum and minimum values. ✓

Find the global maximum and minimum values of $f(x) = 3x - x^2$ on the interval $[0, 4]$.

$f(0) = 0$

$f(4) = 12 - 16 = -4$

$f'(x) = 3 - 2x$

$f'(x) = 0 = 3 - 2x$

$x = 1.5$

$f(1.5)$

$= 3(1.5) - (1.5)^2$

$= 4.5 - 2.25$

$f(1.5) = 2.25$

global max: $(1.5, 2.25)$

" min $(4, -4)$