This Video:

1. Review vectors.

2. Many Example Problems.

① Pause
② Try it
③ unpause it

13.1 DISPLACEMENT VECTORS

Displacement vectors which point in the same direction and have the same magnitude are considered to be the same, even if they do not coincide.

direction -> from magnitude

LA to Buffalo

vector : $\vec{v}$

$\vec{v}$

notation for vector

$\| \vec{v} \|$.

scalar, magnitude of a vector

same starting

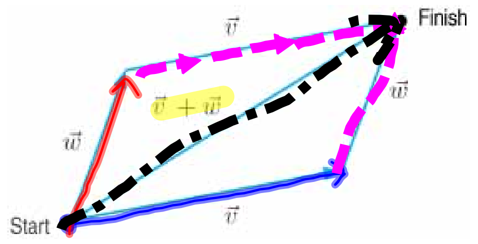
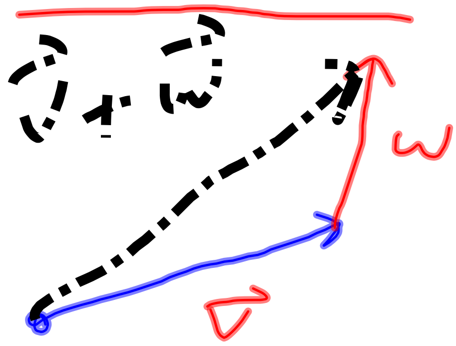


Figure 13.3 The sum $\vec{v}+\vec{w}=\vec{w}+\vec{v}$

tip to tail



Scalar multiplication

- keeps direction *
- magnitude

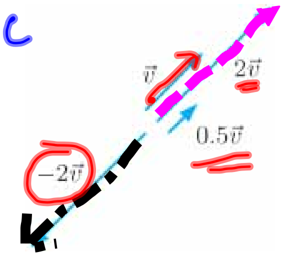


Figure 13.5 Scalar multiples of the vector $\vec{v}$

$\boxed{\lambda \cdot \vec{v}}$

Constant

"landa"

if $\lambda < 0$,
then $\vec{v}$

If λ is a scalar and $\vec{v}$ is a displacement vector, the **scalar multiple of** $\vec{v}$ **by** λ, written $\lambda\vec{v}$, is the displacement vector with the following properties:

- The displacement vector $\lambda\vec{v}$ is parallel to $\vec{v}$, pointing in the same direction if $\lambda > 0$, and in the opposite direction if $\lambda < 0$.
- The magnitude of $\lambda\vec{v}$ is $|\lambda|$ times the magnitude of $\vec{v}$, that is, $\|\lambda\vec{v}\| = |\lambda|\,\|\vec{v}\|$.

$\vec{v}$ $\quad \|\vec{v}\|$

$\lambda\vec{v}$ $\quad \|\lambda\vec{v}\| = |\lambda|\,\|\vec{v}\|$

suppose $\vec{w}$ $\quad \|\vec{w}\| = 10$

$5\vec{w}$ $\quad \|5\vec{w}\| = 5 \cdot 10 = 50$

x, y, z

Components of Displacement Vectors: The Vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$

unit in x   y   z

An Alternative Notation for Vectors

Many people write a vector in 3-dimensions as a string of three numbers, that is, as

$$\boxed{\vec{v} = (v_1, v_2, v_3)} \text{ instead of } \boxed{\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}}.$$

Since the first notation can be confused with a point and the second cannot, we usually use the second form.

velocity in x   vel y   vel z

unit vector : has magnitude equal to 1

Resolve the displacement vector, $\vec{v}$, from the point $P_1 = (2, 4, 10)$ to the point $P_2 = (3, 7, 6)$ into components.

$$\overrightarrow{P_1P_2} = \vec{V} = (3-2)\hat{i} + (7-4)\hat{j} + (6-10)\hat{k}$$

$$\vec{V} = \hat{i} + 3\hat{j} - 4\hat{k}$$

$\lambda \vec{v}$

Decide whether the vector $\vec{v} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ is parallel to each of the following vectors:

$$\vec{w} = 4\vec{i} + 6\vec{j} + 10\vec{k}, \quad \vec{a} = -\vec{i} - 1.5\vec{j} - 2.5\vec{k}, \quad \vec{b} = 4\vec{i} + 6\vec{j} + 9\vec{k}.$$

$\vec{w} = 2\vec{v}$ ☑ parallel

$\vec{a} = -\frac{1}{2}\vec{v}$ ☑ parallel

$\vec{b} \neq \lambda \vec{v}$

$\vec{v}$ λ

$2 \cdot 2 = 4$ ☑

$3 \cdot 2 = 6$ ☑

$5 \cdot 2 \neq 9$ $\varnothing$

not parallel

STOP

vectors

$$\vec{V} = v_1 \hat{\imath} + v_2 \hat{\jmath} + v_3 \hat{\jmath}$$

$$= (v_1, v_2, v_3)$$

$\lambda \vec{v} \rightarrow$ parallel to $\vec{v}$

$\vec{v} + \vec{w}$

MORE ON VECTORS ...

Components of Displacement Vectors

The displacement vector from the point $P_1 = (x_1, y_1, z_1)$ to the point $P_2 = (x_2, y_2, z_2)$ is given in components by

$$\overrightarrow{P_1 P_2} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}.$$

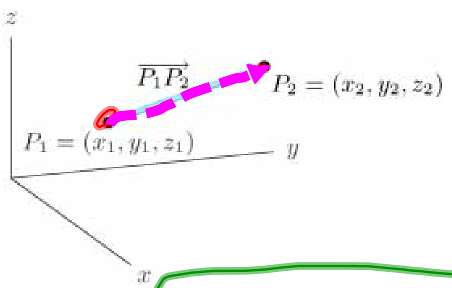


Figure 13.10 The displacement vector $\boxed{\overrightarrow{P_1P_2} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}}$

magnitude = $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Position Vectors: Displacement of a Point from the Origin

A displacement vector whose tail is at the origin is called a *position vector*. Thus, any point (x_0, y_0, z_0) in space has associated with it the position vector $\vec{r}_0 = x_0\vec{i} + y_0\vec{j} + z_0\vec{k}$. (See Figure 13.11.) In general, a position ~~vector gives~~ the displacement of a point from th origin.

(0,0,0)

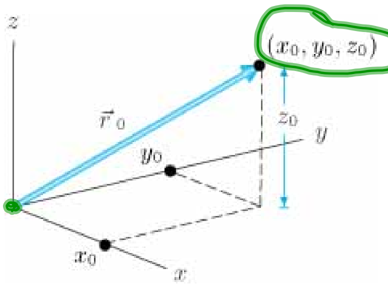


Figure 13.11 The position vector $\vec{r}_0 = x_0\vec{i} + y_0\vec{j} + z_0\vec{k}$

What is the magnitude of the vector r?

[redacted]

$$\|\vec{r}_0\| = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

Summing vectors and Scalar multiplication

$$\vec{v} + \vec{w} = \underline{(v_1 + w_1)}\,\vec{i} + \underline{(v_2 + w_2)}\,\vec{j} + \underline{(v_3 + w_3)}\,\vec{k},$$

$$\lambda \vec{v} = \lambda v_1 \vec{i} + \lambda v_2 \vec{j} + \lambda v_3 \vec{k}.$$

$\vec{v} = 2\vec{i} + 3\vec{j} + 4\vec{k}$

$\vec{w} = 5\vec{i} + 8\vec{j} - \vec{k}$

$\vec{v} + \vec{w} = 7\vec{i} + 11\vec{j} + 3\vec{k}$

$-2\vec{w} = -10\vec{i} - 16\vec{j} + 2\vec{k}$

Unit Vectors

x, y, z

A *unit vector* is a vector whose magnitude is 1. The vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors in the directions of the coordinate axes. It often helpful to find a unit vector in the same direction as a given vector $\vec{v}$. Suppose that $\| \vec{v} \| = 10$; a unit vector in the same directi as $\vec{v}$ is $\vec{v}$ / 10. In general, a unit vector in the direction of any nonzero vector $\vec{v}$ is

$$\vec{u} = \frac{\vec{v}}{\| \vec{v} \|}.$$

$\vec{v}$, $\|\vec{v}\| = 10$

vector of magnitude 1 in the direction of $\vec{v}$

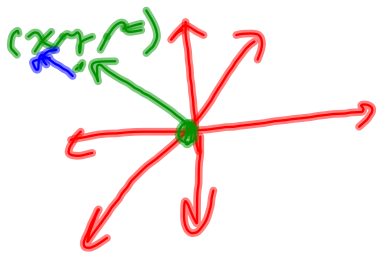
Find a unit vector, $\vec{u}$, in the direction of the vector $\vec{v} = \vec{i} + 3\vec{j}$.

$$\|\vec{v}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\boxed{\vec{u} = \frac{\vec{v}}{\sqrt{10}} = \frac{1}{\sqrt{10}}\vec{i} + \frac{3}{\sqrt{10}}\vec{j}}$$

$\|\vec{u}\| = 1$

Find a unit vector at the point (x, y, z) that points radially outward away from the origin.



$$\vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\|\vec{v}\| = \sqrt{x^2+y^2+z^2}$$

$$\vec{u} = \frac{1}{\sqrt{x^2+y^2+z^2}}\left[x\vec{i}+y\vec{j}+z\vec{k}\right]$$

The vector from the origin to (x, y, z) is the position vector

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}.$$

Thus, if we put its tail at (x, y, z) it will point away from the origin. Its magnitude is

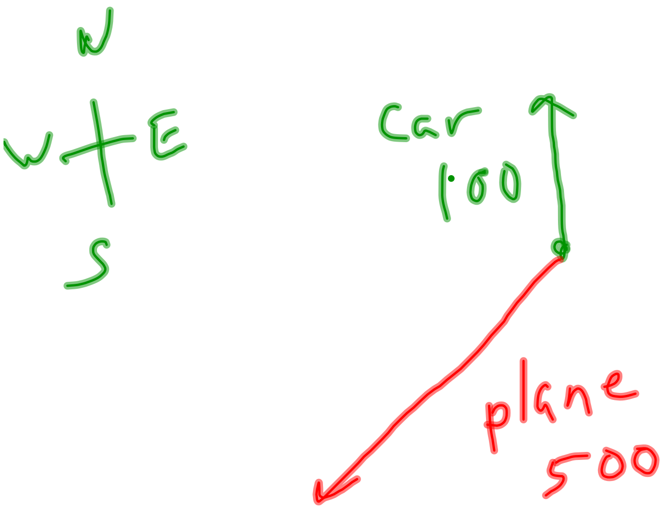
$$\|\vec{r}\| = \sqrt{x^2+y^2+z^2},$$

so a unit vector pointing in the same direction is

$$\frac{\vec{r}}{\|\vec{r}\|} = \frac{x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}\vec{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}.$$

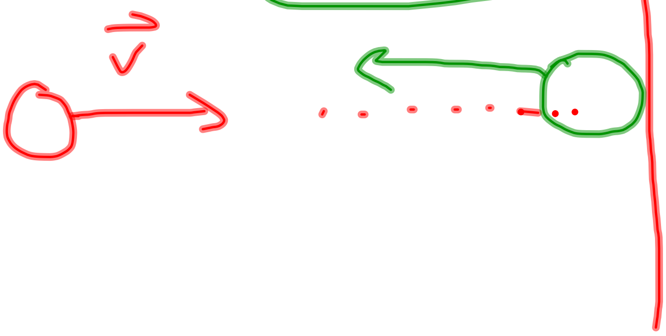
The **velocity vector** of a moving object is a vector whose magnitude is the speed of the object and whose direction is the direction of its motion.

A car is traveling north at a speed of 100 km/hr, while a plane above is flying horizontally south-west at a speed of 500 km/hr. Draw the velocity vectors of the car and the plane.



A ball is moving with velocity $\vec{v}$ when it hits a wall at a right angle and bounces straight back, with its speed reduced by 20%. Express its new velocity in terms of the old one.

$$\vec{V}_{NEW} = -0.8\vec{V}$$



Commutativity

$x+y=y+x$

1. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

Associativity

2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$

Distributivity

4. $(\alpha+\beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$
5. $\alpha(\vec{v} + \vec{w}) = \alpha\vec{v} + \alpha\vec{w}$

Identity vector

6. $1\vec{v} = \vec{v}$
7. $0\vec{v} = \vec{0}$
8. $\vec{v} + \vec{0} = \vec{v}$
9. $\vec{w} + (-1)\vec{v} = \vec{w} - \vec{v}$

Suppose the vector $\vec{I}$ represents the number of copies, in thousands, made by each of four copy centers in the month of December and $\vec{J}$ represents the number of copies made at the same four copy centers during the previous eleven months (the "year-to-date"). If $\vec{I} = (25, 211, 818, 642)$, and $\vec{J} = (331, 3227, 1377, 2570)$, compute $\vec{I} + \vec{J}$. What does this sum represent?

$$\vec{I} = (25, 211, 818, 642)$$

# of copies in the all year

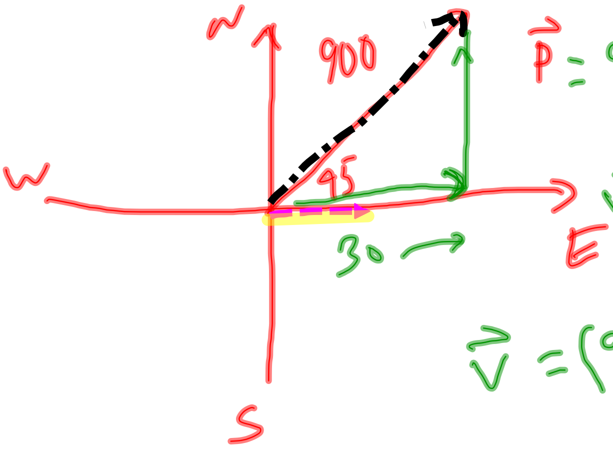
$$\overrightarrow{I+J} = (356, 3268, 2195, 3212)$$

Example Problems

Chapter 13, Section 13.2, Question 24

An airplane heads northeast at an airspeed of 900 km/hr, but there is a wind blowing from the west at 30 km/hr.

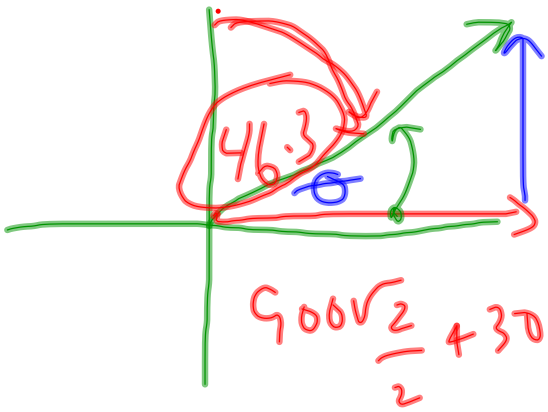
In what direction does the plane end up flying? Give the angle relative to north and to one decimal place.



$\vec{P} = 900\cos 45\,\vec{i} + 900 \cdot \sin 45\,\vec{j}$

$\vec{W} = 30\,\vec{i}$

$\vec{V} = \left(900\frac{\sqrt{2}}{2} + 30\right)\vec{i} + 900\sin 45\,\vec{j}$



$$\tan^{-1}\left(\frac{900\sqrt{2}/2}{900\frac{\sqrt{2}}{2} + 30}\right) = \theta$$

$\theta = 43.7$

46.3° E of N

cancel out $\vec{j}$

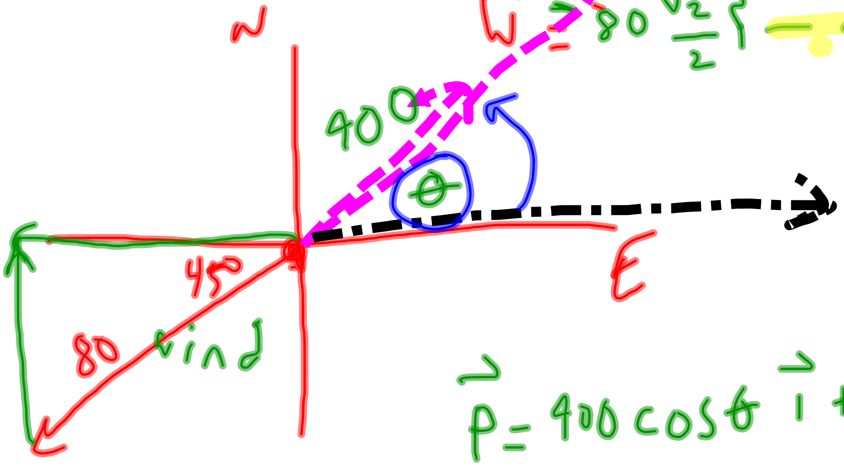
Chapter 13, Section 13.2, Question 15

An airplane is flying at an airspeed of 400 km/hr in a cross-wind that is blowing from the northeast at a speed of 80 km/hr. In what direction should the plane head to end up going due east?

plane

Let φ be the angle from the axis which points east to the velocity of the airplane, relative to the air.

$$\vec{W} = 80\frac{\sqrt{2}}{2}\vec{i} - 80\frac{\sqrt{2}}{2}\vec{j}$$



$$\vec{P} = 400\cos\theta\,\vec{i} + 400\sin\theta\,\vec{j}$$

$$-\frac{80\sqrt{2}}{2} + 400\sin\theta = 0 \; ; \text{ no } \vec{j}$$

$$\sin\theta = \frac{80\sqrt{2}}{800} = \frac{\sqrt{2}}{10}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{10}\right)$$

$$\theta = 8.13$$

Chapter 13, Section 13.1, Question 29

Find the value(s) of a making $\vec{v} = 2a\,\vec{i} - 8\,\vec{j}$ parallel to $\vec{w} = a^2\,\vec{i} + 24\,\vec{j}$.

$\times(-3)$

Enter your answers in increasing order of a.

$(-3)\,\vec{v} = \vec{w}$

$-3\vec{v} = -6a\,\vec{i} + 24\,\vec{j}$

$\vec{w} = a^2\,\vec{i} + 24\,\vec{j}$

$a^2 = -6a$

$a^2 + 6a = 0$

$a(a+6) = 0$

$\boxed{a = -6, 0}$

Chapter 13, Section 13.1, Question 18

Find the length of the vector.

$$\vec{v} = 8.8\ \vec{i} - 3.1\ \vec{j} + 3.8\ \vec{k}$$

Round your answer to one decimal place.

$$\|\vec{v}\| = \sqrt{8.8^2 + (-3.1)^2 + (3.8)^2}$$

Chapter 13, Section 13.1, Question 09

Perform the indicated computation.

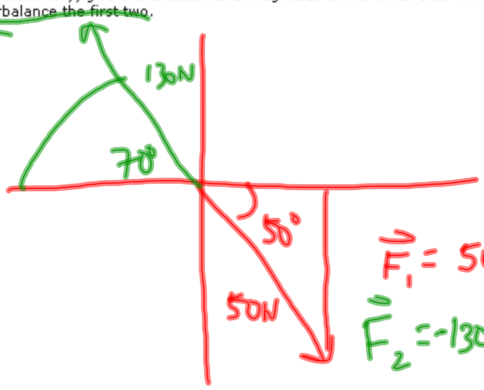
$$-9\left(\vec{i} - 5\,\vec{j}\right) - 0.5\left(\vec{i} - \vec{k}\right) =$$

$$-9\vec{i} + 45\vec{j} - .5\vec{i} + .5\vec{k}$$

$$-9.5\vec{i} + 45\vec{j} + .5\vec{k}$$

Chapter 13, Section 13.2, Question 22

One force is pushing an object in a direction 50° south of east with a force of 50 newtons. A second force is simultaneously pushing the object in a direction 70° north of west with a force of 130 newtons. If the object is to remain stationary, give the direction and magnitude of the third force which must be applied to the object to counterbalance the first two.



Total $\vec{F} = 0$

$$\vec{F}_1 = 50\cos 50\,\vec{i} - 50\sin 50\,\vec{j}$$

$$\vec{F}_2 = -130\cos 70\,\vec{i} + 130\sin 70\,\vec{j}$$

$$\overrightarrow{F_1 + F_2} = -12.3\vec{i} + 83.9\vec{j}$$

$$\vec{F}_3 + \vec{F}_{1+2} = 0$$

$$\vec{F}_3 = 12.3\vec{i} - 83.9\vec{j}$$

$$\|\vec{F}_3\| = \sqrt{12.3^2 + 83.9^2} = 84.8$$

$$\theta = \tan^{-1}\left(\frac{83.9}{12.3}\right) = 81.7^\circ \text{ S of E}$$