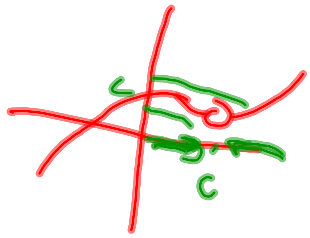
$\lim_{x \to c} f(x) = L$

LIMITS



The function f has a **limit** L at the point (a, b), written

$$\lim_{(x,y)\to(a,b)} f(x, y) = L,$$

if $f(x, y)$ is as close to L as we please whenever the distance from the point (x, y) to the point (a, b) is sufficiently small, but not zero.

$f(a,b) =$ exists

A function f is **continuous at the point** (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

A function is **continuous on a region** R in the xy-plane if it is continuous at each point in R.

CONTINUOUS: CALCULUS DEFINITION

#1 $f(a,b)$ exists

#2 $\lim_{(x,y)\to(a,b)} f(x,y)$ exists

#3 $f(a,b) = \lim_{(x,y)\to(a,b)} f(x,y)$

$$f(x,y) = \frac{x^2 y}{x^2 + y^2}$$

(a) $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

① FIX "Y", LET X APPROACH A. SET Y EQUAL TO B

② FIX "X", LET Y APPROACH B. SET X EQUAL TO A.

③ THESE LIMITS MUST EQUAL EACH OTHER. ✓

① $\frac{0^2 y}{0^2 + y^2} = \frac{0}{y^2}$

$\lim_{y\to 0} \frac{0}{y^2} = 0$

② $\lim_{x\to 0} \frac{x^2(0)}{x^2 + 0^2} = 0$

(b) $\lim_{(x,y)\to(0,0)} g(x,y)$ does not exist.

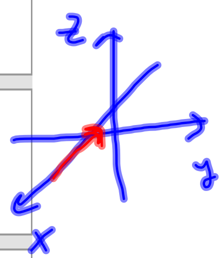
$$g(x,y) = \frac{x^2}{x^2+y^2}.$$

$$x: \lim_{x\to 0} \frac{x^2}{x^2+0^2} = 1$$

$$y: \lim_{y\to 0} \frac{0^2}{0^2+y^2} = 0$$

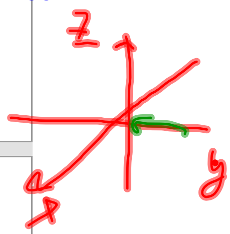
Consider the function $f(x, y) = \dfrac{x - 2y^2}{x + 2y^2}$.

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{x-2y^2}{x+2y^2}$ as (x, y) approaches the origin along the positive x -axis.



$$\lim_{x\to 0} \frac{x-0}{x-0} = 1$$

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{x-2y^2}{x+2y^2}$ as (x, y) approaches the origin along the positive y -axis.



$$\lim_{y\to 0} \frac{-2y^2}{2y^2} = -1$$

Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist?

- ○ yes
- ◉ no

Is $f(x, y)$ continuous at $(0, 0)$?

- ◉ no
- ○ yes

Chapter 12, Section 12.6, Question 18

Determine whether there is a value for C making the folowing function continuous everywhere.

$$f(x, y) = \begin{cases} C + y & x \le 9 \\ 10 - x & x > 9 \end{cases}$$

- 9
- 19
- 90
- (selected) There is no value for C
- 10

@ $x = 9$

$C + y = 10 - 9$

$C + y = 1$

$C = 1 - y$

Chapter 12, Section 12.6, Question 07

Find the limits of the function $f(x, y)$ as $(x, y) \to (0,0)$.

$$f(x, y) = 2e^{-5x-6y}$$

- ○ $2e^{-11}$
- ◉ 2
- ○ $2e^{11}$
- ○ 0
- ○ 1

$x=0$

$\lim_{y\to 0} 2e^{-5(0)-6y}$

$= 2\boxed{e^{-5(0)-6(0)}}$

$=2$

$y=0$

$\lim_{x\to 0} 2e^{-5x-0}$

$=2e^{-0-0}$

$=2$