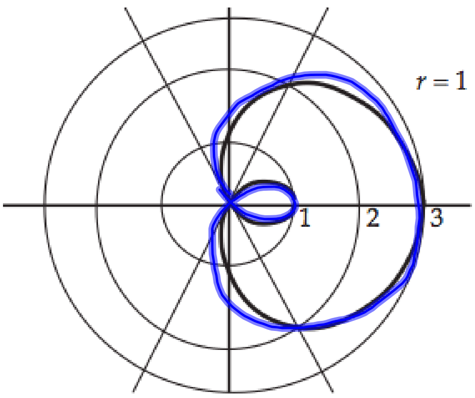
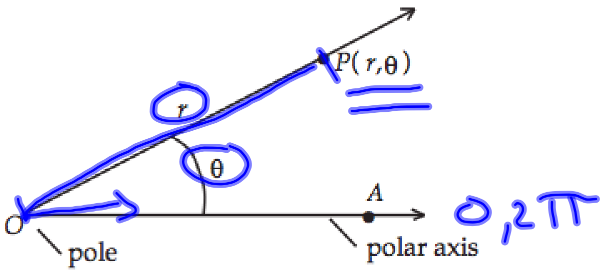
Polar Coordinates

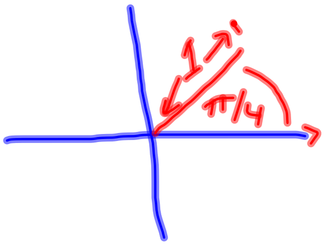
- length of radius from origin
- angle of radius

r, θ

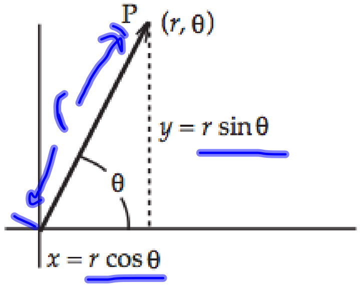




$(1, \pi/4)$



$$(x,y) = (r\cos\theta, r\sin\theta) \text{ or } \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$ ✓



$x^2 = r^2\cos^2\theta$

So what is the relationship between x and y?

$y^2 = r^2\sin^2\theta$

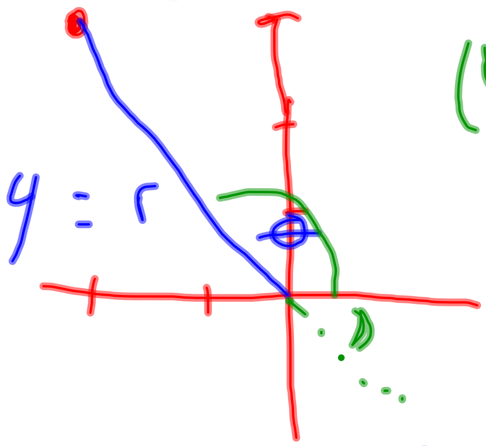
And what about y/x?

$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta)$

$x^2 + y^2 = r^2$ ✓

$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan(\theta)$ ✓

Find polar coordinates of the point with Cartesian coordinates $(-2, 2\sqrt{3})$.



$$(r, \theta) = \left(4, \frac{2\pi}{3}\right)$$

$$\tan\theta = \frac{2\sqrt{3}}{-2}$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$= -\pi/3$$

$$\theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$r^2 = (-2)^2 + (2\sqrt{3})^2$$

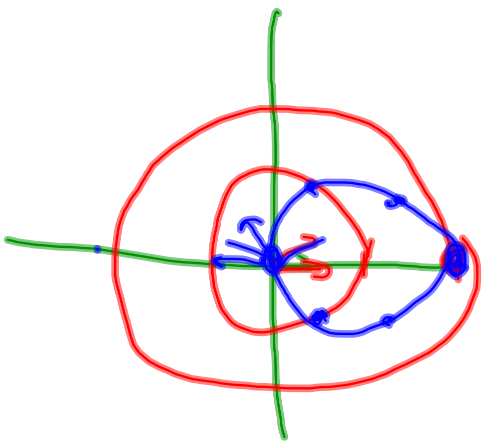
$$r = \sqrt{4+12}$$

$$\boxed{r = 4}$$

Sketch a graph of the set of points with polar coordinates equation $r = 2\cos\theta$.

(sol'n on next slide)

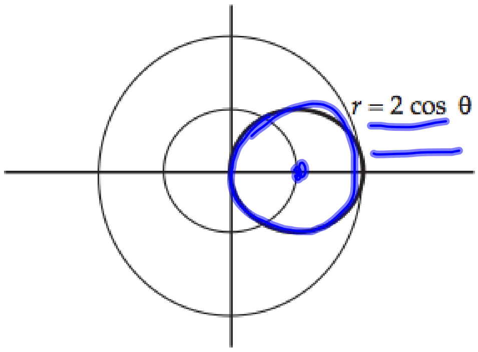
θ	r
0	$2\cos(0) = 2$
$\pi/6$	$2\cos(\pi/6) = \sqrt{3}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$5\pi/6$	$-\sqrt{3}$
π	-2



We make a table of inputs θ and outputs $r = 2\cos\theta$ and use the pairs to plot points.

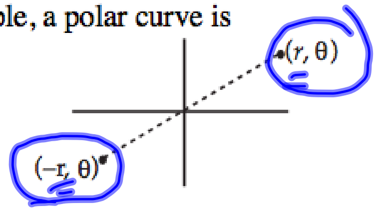
θ	r
0	2
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-1
$\frac{5\pi}{6}$	$-\sqrt{3}$
π	-2

θ	r
$\frac{7\pi}{6}$	$-\sqrt{3}$
$\frac{4\pi}{3}$	-1
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	1
$\frac{11\pi}{6}$	$\sqrt{3}$
2π	2



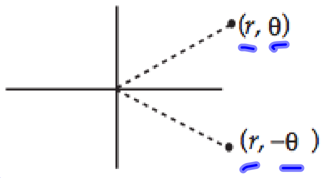
Symmetry

Three tests for symmetry in polar graphs are useful. For example, a polar curve is

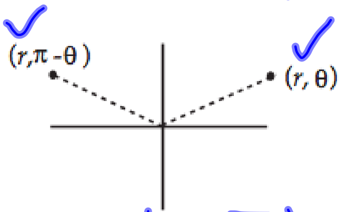


a) symmetric about the origin if substitution of $-r$ for r produces an equivalent equation;

b) symmetric about the x-axis if substitution of $-\theta$ for θ produces an equivalent equation;



c) symmetric about the y-axis if substitution of $\pi - \theta$ for θ produces an equivalent equation.



$\cos\left(\frac{\pi}{4}\right) = \sqrt{2}/2$

$\cos\left(-\frac{\pi}{4}\right) = \sqrt{2}/2$

$\sin\left(\frac{\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right)$

$\boxed{= 1/2}$

Sketch a graph of the polar curve $\boxed{r = 1 - \cos\theta}$.

USE SYMMETRY!!

(sol'n next page)

θ	r
0	0
$\pi/6$	$1 - \sqrt{3}/2$
$\pi/3$	$1 - \frac{1}{2} = 1/2$
$\pi/2$	$1 - 0 = 1$
$2\pi/3$	$1 - (-\frac{1}{2}) = 1.5$

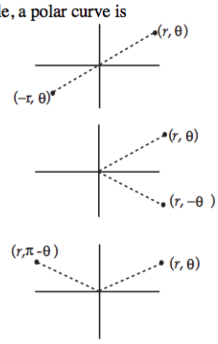
Symmetry ✓

Three tests for symmetry in polar graphs are useful. For example, a polar curve is

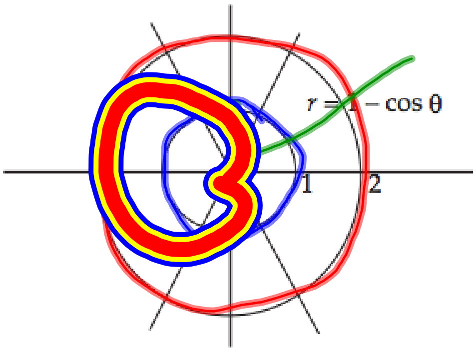
a) symmetric about the origin if substitution of $-r$ for r produces an equivalent equation;

b) symmetric about the x-axis if substitution of $-\theta$ for θ produces an equivalent equation;

c) symmetric about the y-axis if substitution of $\pi - \theta$ for θ produces an equivalent equation.



θ	r
0	0
$\frac{\pi}{6}$	$1-\frac{\sqrt{3}}{2} \approx .13$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
$\frac{5\pi}{6}$	$1+\frac{\sqrt{3}}{2} \approx 1.87$
π	2



This heart-shaped curve is called a **cardioid**, from the Greek (kordia), meaning heart.

USING A GRAPHING CALCULATOR