Motion in a Plane

Vectors

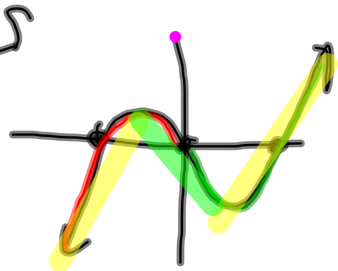
position

let $s(t)=t^3-t$

find v(t). $3t^2-1$

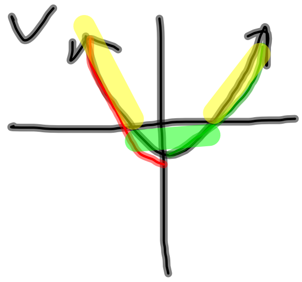
Find a(t) $6t$

s



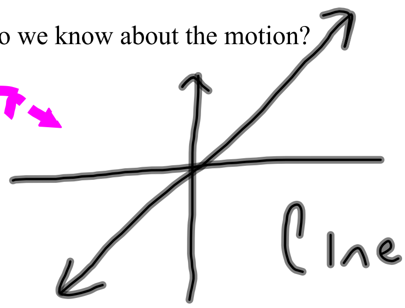
cubic

v



quadratic

What do we know about the motion?

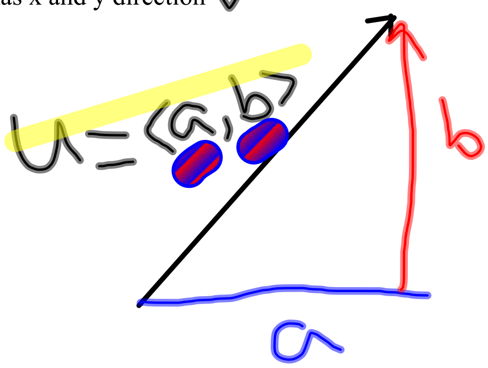


linear

Using vectors: direction and magnitude

$\langle x, y \rangle$

-name a vector ✓
-has x and y direction ✓



Write this in terms of x and y.

$\boldsymbol{u} = \langle a, b \rangle$

$u = \langle a, b \rangle$

Vector Addition

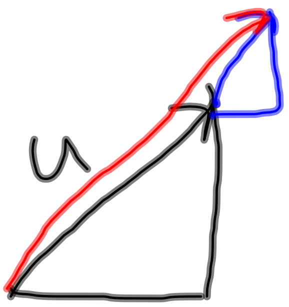
If $\boldsymbol{u} = \langle a, b \rangle$ and $\boldsymbol{v} = \langle c, d \rangle$, then the vector $\boldsymbol{u} \oplus \boldsymbol{v}$ is defined by

$$\boldsymbol{u} \oplus \boldsymbol{v} = \langle a, b \rangle \oplus \langle c, d \rangle = \langle a + c, b + d \rangle.$$

$$\langle 3, 4 \rangle + \langle 1, 2 \rangle$$

$$= \langle 3+1, 4+2 \rangle$$

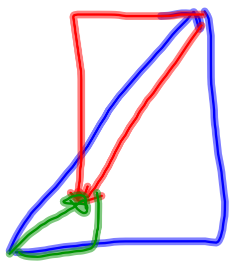
$$= \langle 4, 6 \rangle$$



$2 + 4$ 6

$3 + 1 = 4$

$\langle 3, 4 \rangle + \langle -2, -3 \rangle$



$\langle 3 + -2, \ 4 + -3 \rangle$

$= \langle 1, 1 \rangle$

Multiplication of a Vector by a Real number

If r is a real number and $\boldsymbol{u} = \langle a, b \rangle$, then the vector $r \cdot \boldsymbol{u}$ is defined by

$$r \cdot \boldsymbol{u} = \langle ra, rb \rangle.$$

Examples with graphs:

$r \cdot u$

$r = 2$

$u = \langle 3, 7 \rangle$

$ru = \langle 2 \cdot 3, 2 \cdot 7 \rangle$

$= \langle 6, 14 \rangle$

The Standard Unit Vectors *i* and *j* unit - length 1

Two vectors that play an important role in V_2 are given special names:

- $\boxed{i = \langle 1, 0 \rangle}$ and $\boxed{j = \langle 0, 1 \rangle}$.

x y

Write <-3, 7> in terms of *i* and *j*.

= <-3,0> + <0,7>
=-3<1,0> + 7<0,1>

=-3i+7j

Write the following vectors in terms *i* and *j*.

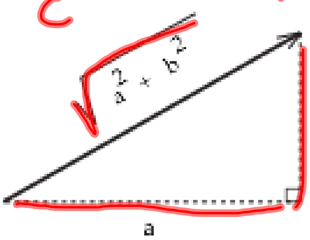
< 2,5 > 2i + 5j

< 3,-2 > 3i - 2j

Length of a Vector

The length of an arrow which represents a vector $\langle a, b\rangle$ is $\sqrt{a^2+b^2}$.

c $\quad a^2+b^2=c^2$



Thus we have

> **Length of a Vector**
> The length of a vector $\boldsymbol{u} = \langle a, b\rangle$, denoted by $|\ \boldsymbol{u}\ |$, is defined by,
>
> $$|\ \boldsymbol{u}\ | = \sqrt{a^2+b^2}$$

u

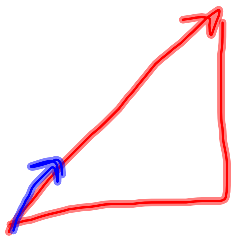
length of u = $|u|$

Unit Vector: A vector with length 1.

Find a unit vector in the same direction as <3,4>.

has length = 1

$$|\langle 3,4\rangle| = \sqrt{9+16} = 5$$

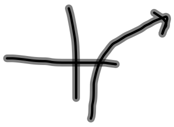


$$\frac{u}{|u|} \rightarrow$$ unit vector in the same direction

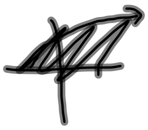
$$\frac{\langle 3,4\rangle}{5} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

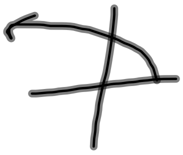
What is the domain of the following function?

$$\boldsymbol{R}(t) = (\ln t) \cdot \boldsymbol{i} \oplus \sqrt{1-t} \cdot \boldsymbol{j}$$

$\ln(t)$  D: $t > 0$ ✓

The domain is the intersection of the respective component domains.

$\sqrt{1-t}$  D: $t \leq 1$ ✓

 $\boxed{D: 0 < t \leq 1}$

1. If $\boldsymbol{u} = \langle -1,\ 2 \rangle$, sketch the arrows that represent

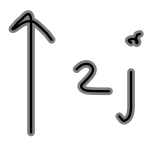
a) $\boldsymbol{u} \oplus \boldsymbol{i}$ b) $\boldsymbol{u} \oplus \boldsymbol{j}$ c) $\boldsymbol{u} \oplus \boldsymbol{i} \oplus \boldsymbol{j}$

d) $-\boldsymbol{j}$ e) $2\boldsymbol{u} \ominus 2\boldsymbol{j}$

a) $u \oplus i$

$\langle -1, 2 \rangle \oplus \langle 1, 0 \rangle$

$= \langle 0, 2 \rangle$



b) $u \oplus j$

$\langle -1, 2 \rangle \oplus \langle 0, 1 \rangle$

$= \langle -1, 3 \rangle$

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$$c = \langle -1, 2 \rangle \oplus \langle 1, 0 \rangle \oplus \langle 0, 1 \rangle$$

$$= \langle 0, 3 \rangle \quad \uparrow 3$$

$$= 3j$$

d $\quad -\langle 0, 1 \rangle = \langle 0, -1 \rangle = \downarrow 1$

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$$2\langle -1, 2 \rangle - 2\langle 0, 1 \rangle$$

$$= \langle -2, 4 \rangle - \langle 0, 2 \rangle$$

$$= \langle -2, 2 \rangle = -2i + 2j$$

$=$