if $y' = ky\left(1 - \frac{y}{L}\right)$

then $y = \frac{L}{1 + A \cdot e^{kx}}$

where $A = \frac{L - y_0}{y_0}$

Solve $\boxed{y' = 0.08y\left(1 - \dfrac{y}{1000}\right); \quad y(0) = 100}$

$$y' = ky\left(1 - \frac{y}{L}\right) \longrightarrow y = \frac{L}{1 + A\cdot e^{-kx}}$$

$$y' = 0.08y\left(1 - \frac{y}{1000}\right)$$

where

$$A = \frac{L - y_0}{y_0}$$

L = limiting amount → max (y)

$L = 1000$ ✓

$k = .08$ ✓

$y_0 = 100$

$A = \dfrac{1000 - 100}{100} = 9$ ✓

$$y = \frac{1000}{1 + 9e^{-0.08x}}$$

Given $\boxed{f(x) = \dfrac{50}{1+4e^{-.2x}}}$

Find $\boxed{\lim_{x \to -\infty} f(x)} = \underline{\underline{0}}$.

$$= \frac{50}{1+4e^{(-.2)(-\infty)}} \approx \frac{50}{1+4(\infty)}$$

$$\approx \frac{50}{\text{Huge}}$$

$$\approx 0$$

Given $f(x) = \dfrac{50}{1+4e^{-.2x}}$

(b) Find

$$\boxed{\lim_{x\to+\infty} f(x) = 50}$$

function approach limiting value

$$= \frac{50}{1+4e^{-.2(\infty)}}$$

$$= \frac{50}{1+4e^{-\infty}} = \frac{50}{1+\cancel{4\cdot\frac{1}{e^{\infty}}}}$$

0

$$= \frac{50}{1} = 50$$

$y' = ky\left(1-\frac{y}{L}\right) \longleftrightarrow y = L/1+Ae^{-kx}$

Given $f(x) = \frac{50}{1+4e^{-.2x}}$

$50 \rightarrow L$

$-.2 \rightarrow k$

(c) find the point of inflection

$y''=0$

$$y' = -0.2y\left(1-\frac{y}{50}\right)$$

$$y' = -0.2y + \frac{0.2y^2}{50}$$

$$y'' = -0.2 + \frac{0.4y}{50}$$

$$0 = -.2 + \frac{0.4y}{50}$$

$$0.2 = 0.4y/50$$

$$\boxed{y=25} \rightarrow \text{Point of Inflection}$$

$$f(x) = \frac{50}{1+4e^{-.2x}}$$

when $y = 25$, POI $\to$ what x equal?

$$25 = \frac{50}{1+4e^{-.2x}}$$

$$\frac{50}{2} = \frac{50}{1+4e^{-.2x}}$$

$$2 = 1+4e^{-.2x}$$

$$1 = 4e^{-.2x}$$

$$\frac{1}{4} = e^{-.2x}$$

$$\ln\left(\frac{1}{4}\right) = -.2x$$

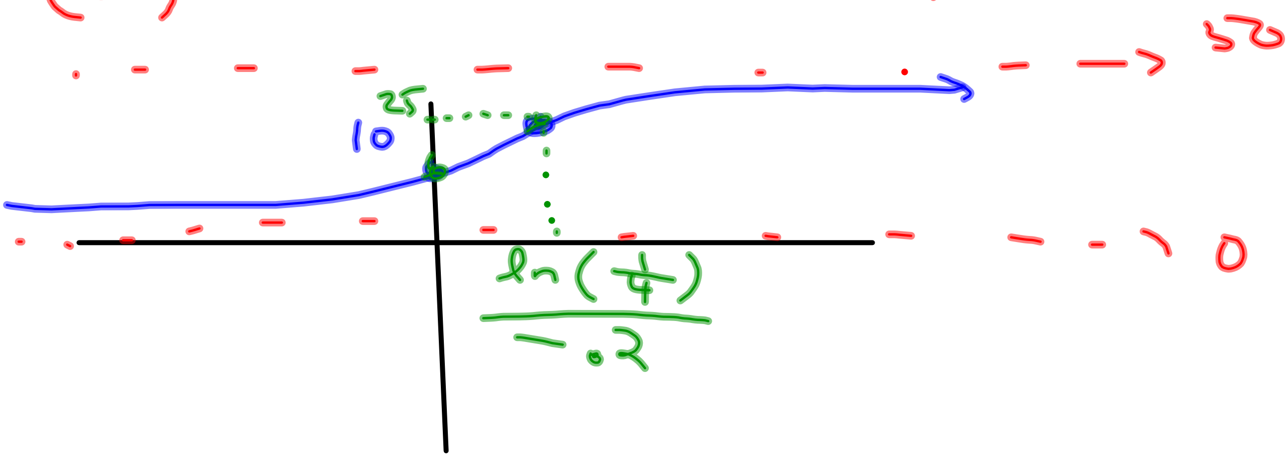
$$\boxed{x = \frac{\ln\left(\frac{1}{4}\right)}{-.2}}$$

Given $f(x) = \dfrac{50}{1+4e^{-.2x}}$

(d) Sketch $f(x)$

$x = 0$

$f(0) = \dfrac{50}{1+4(1)}$



$\boxed{\text{General}}$ Given

$$* \quad \boxed{y' = ky\left(1-\frac{y}{L}\right)}, \text{ find POI for } y$$

$$y' = ky - \frac{ky^2}{L}$$

$$* \quad y'' = k - \frac{2ky}{L}$$

$$* \quad 0 = k - \frac{2ky}{L}$$

$$* \quad \frac{2ky}{L} = k$$

$$\boxed{y = \frac{L}{2}}$$

Find x

POI when $y = \frac{L}{2}$

$\cancel{f}$ $$\boxed{y = \frac{L}{1 + Ae^{-kx}}}$$

Find x POI

$$\frac{L}{2} = \frac{L}{1+Ae^{-kx}}$$

$$2 = 1 + Ae^{-kx}$$

$$1 = Ae^{-kx}$$

$$\frac{1}{A} = e^{-kx}$$

$$\ln\left(\frac{1}{A}\right) = -kx$$

$$x = \frac{\ln\left(\frac{1}{A}\right)}{-k}$$

POI:

$$\left(\frac{\ln\left(\frac{1}{A}\right)}{-k}, \frac{L}{2}\right)$$

Logistic Model

$$y' = ky\left(1-\frac{y}{L}\right)$$

$$y = \frac{L}{1+Ae^{-kx}}, \quad \text{for } A = \frac{L-y_0}{y_0}$$

$\lim_{x\to-\infty} y = 0$	; $\lim_{x\to\infty} y = L$
<u>y int</u> $x=0$ $= \frac{L}{1+Ae^{0}}$ $= \frac{L}{1+A}$	POI $\left(\frac{\ln\left(\frac{1}{A}\right)}{-k}, \frac{L}{2}\right)$