

* Find

$$\int \frac{x^3}{1+x^2} dx$$

Partial
Fractions

* $x^2 + 0x + 1$

$$\begin{array}{r} x - \frac{x}{x^2+1} \\ \hline x^3 + 0x^2 + 0x + 0 \\ - (x^3 + 0x^2 + 1x) \\ \hline -x \end{array}$$

$$\int \left(x - \frac{x}{x^2+1} \right) dx$$

$$\int x dx - \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$= \frac{1}{2} x^2 - \frac{1}{2} \ln |x^2+1| + C$$

Evaluate $\int_3^4 \frac{2}{x^2-2x} dx$ *

$$\frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$2 = (x-2)A + xB$$

$$2 = Ax - 2A + Bx$$

$$x=0$$

$$2 = -2A$$

$$A = -1$$

$$x=2$$

$$2 = 2B$$

$$B = 1$$

$$\int_3^4 \left(-\frac{1}{x} + \frac{1}{x-2} \right) dx$$

$$-\ln(x) + \ln(x-2)$$

$$-\ln 4 + \ln 3 + \ln(2) - \cancel{\ln(1)}$$

$$\ln\left(\frac{3 \cdot 2}{4}\right)$$

$$= \ln\left(\frac{3}{2}\right)$$