

## 10.3 Partial Fractions

$$* \left[ \frac{2}{x+3} + \frac{3}{x+1} \right] \cdot$$

$$\frac{(x+1)2}{(x+1)(x+3)} + \frac{3(x+3)}{(x+1)(x+3)}$$

$$= \frac{2x+2+3x+9}{(x+1)(x+3)}$$

$$= \frac{5x+11}{(x+1)(x+3)}$$

Work the other  
direction

$$\frac{3x}{x^2+x-2} =$$

① Factor

② Split  
the  
factors

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

③ Find  
A/B

$$\frac{3x}{(x+2)(x-1)} = \frac{A(x-1)}{(x-1)(x+2)} + \frac{B(x+2)}{(x-1)(x+2)}$$

$$3x + 0 = Ax - A + Bx + 2B$$

$$3x = \underline{A}x + Bx$$

$$3x = 2Bx + Bx$$

$$3x = 3Bx$$

$$B = 1$$

$$0 = -A + 2B$$

$$A = \underline{2B}$$

$$A = 2$$

$$\frac{3x}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$$

$$\int \frac{3x}{x^2+x-2} dx$$

$$= \int \left( \frac{2}{x+2} + \frac{1}{x-1} \right) dx$$

$$= \int \frac{2}{x+2} dx + \int \frac{1}{x-1} dx$$

$$= 2 \cdot \ln|x-2| + \ln|x-1| + C$$

$$= \ln|x-2|^2 + \ln|x-1| + C$$

$$= \ln \left| |x-2|^2 |x-1| \right| + C$$

Find  $\int \frac{x^2 - x + 4}{x^3 - 3x^2 + 2x} dx$

$$= \frac{x^2 - x + 4}{x(x-2)(x-1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

$$\frac{x^2 - x + 4}{x(x-2)(x-1)} = \frac{A(x-1)(x-2) + B(x)(x-2) + C(x)(x-1)}{x(x-2)(x-1)}$$

$X=0$

$$4 = A(-1)(-2)$$

$$4 = 2A$$

$$A=2$$

$X=2$

$$6 = C(2)(1)$$

$$C=3$$

$X=1$

$$4 = B(1)(-1)$$

$$B = -4$$

$$\int \left[ \frac{2}{x} + \frac{-4}{x-1} + \frac{3}{x-2} \right] dx$$

$$\int \frac{2}{x} dx + \int \frac{3}{x-2} dx - \int \frac{4}{x-1} dx$$

$$= 2 \ln|x| + 3 \ln|x-2| - 4 \ln|x-1| + C$$

$$= \ln \left( \frac{|x|^2 |x-2|^3}{|x-1|^4} \right) + C$$