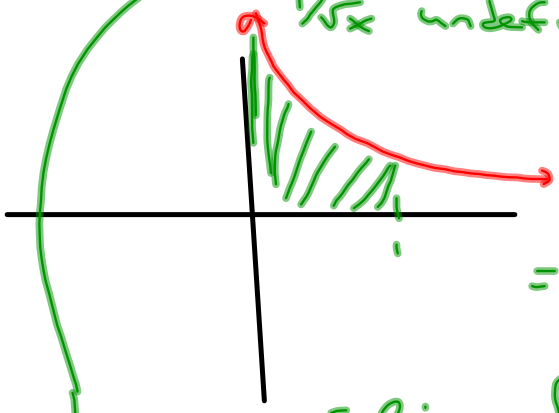
10.2 Part 2

Is $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ ✓Convergent or ✓Divergent?



$= \lim_{a \to 0} \int_a^1 \frac{1}{\sqrt{x}}\,dx$ ☑

$= \lim_{a \to 0} \int_a^1 x^{-1/2}\,dx$

$= \lim_{a \to 0} \left[2x^{1/2} \Big|_a^1 \right]$

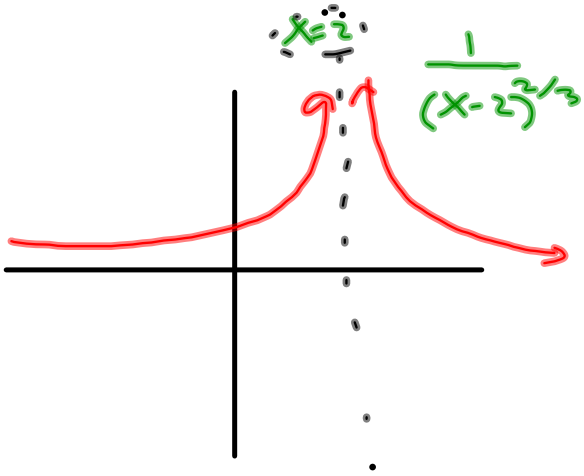
$= \lim_{a \to 0} \left[2\sqrt{1} - 2\sqrt{a} \right]$

$= 2\sqrt{1} - 0$

$= 2$

CONVERGENT!

Do $\int_1^2 (x-2)^{-2/3}\,dx$ converge or diverge?



$$= \lim_{b \to 2} \int_1^b 1(x-2)^{-2/3}\,dx$$

$$= \lim_{b \to 2} 3(x-2)^{1/3} \Big|_1^b$$

$$= \lim_{b \to 2} \left[3(b-2)^{1/3} - 3(1-2)^{1/3}\right]$$

$$= \left[0 - 3(-1)^{1/3}\right.$$

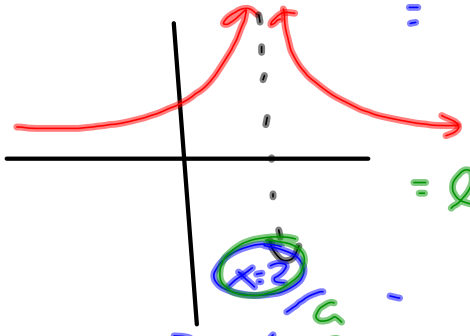
$$= 0 - 3(-1)$$

$$= 3$$

converges !!

Does $\int_1^4 (x-2)^{-2/3}\,dx$ converge or diverge?

$f(x)$



$$= \int_1^2 (x-2)^{-2/3}\,dx + \int_2^4 (x-2)^{-2/3}\,dx$$

$$= \lim_{a\to 2}\left[\int_1^a f(x)\,dx + \int_a^4 f(x)\,dx\right]$$

$$= \lim_{a\to 2}\int_1^a f(x)\,dx + \lim_{a\to 2}\int_a^4 f(x)\,dx$$

$$= 3 + \lim_{a\to 2}\int_a^4 (x-2)^{-2/3}\,dx$$

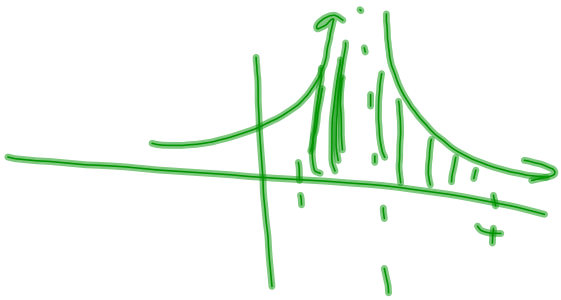
$$= 3 + \lim_{a\to 2} 3[x-2]^{1/3}\Big|_a^4$$

$$= 3 + 3\lim_{a\to 2}\left[(4-2)^{1/3} - (a-2)^{1/3}\right]$$

$$= 3 + 3\left[2^{1/3} - 0\right]$$

$$= 3 + 3\sqrt[3]{2}$$

converges



$\int_1^4 (x-2)^{-2/3}\,dx =$ Area $= 3 + 3\sqrt[3]{2}$