10.2 Improper Integrals: Convergent or divergent

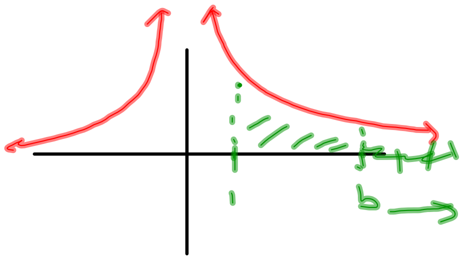
convergent: going to a specific #

divergent: going on to ∞

Consider:

$$\boxed{\int_1^b \frac{1}{x^2}\,dx}$$

$b>1$



$$\lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx$$

$$= \lim_{b\to\infty}\int_1^b x^{-2}\,dx$$

$$= \lim_{b\to\infty}\left[-x^{-1}\right]_1^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{b} - -\frac{1}{1}\right)$$

$$= \left[-\frac{1}{\infty}+1\right]$$

$$\boxed{=1}$$

$$\int_1^\infty \frac{1}{x^2}\,dx = \boxed{1}$$

convergent

Definitions

* if f is is continuous on $[a, \infty)$, $\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$

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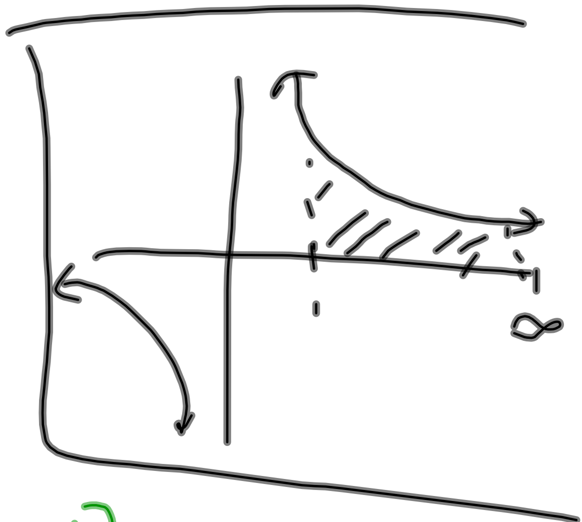
* if f is continuos on $(-\infty, b]$ then, $\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$

Improper integrals are

- convergent if Limit Exists
- divergent if Limit DNE.

is $\displaystyle\int_1^{\infty} \frac{1}{x}\,dx$ convergent or divergent?



$$= \lim_{b\to\infty} \int_1^{b} \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \Big[\ln|x|\Big]_1^{b}$$

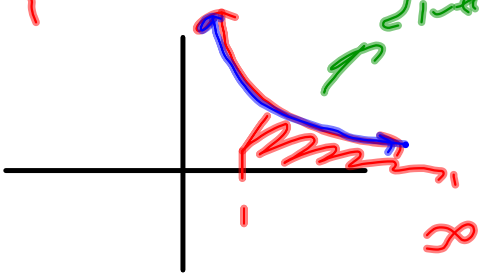
$$= \lim_{b\to\infty} \Big(\ln|b| - \underline{\ln|1|}\Big)$$

$$= \ln|\infty| - 0$$

goes on forever

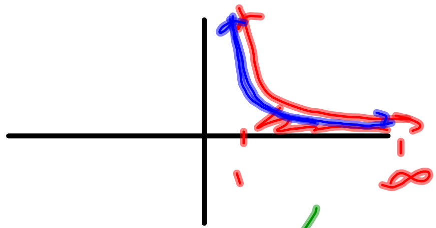
$$\rightarrow \int_1^{\infty} \frac{1}{x}\,dx \text{ is } \boxed{\text{divergent}}$$

$$\int_1^\infty \frac{1}{x}\,dx$$



divergent

$$\int_1^\infty \frac{1}{x^2}\,dx$$



convergent to 1

why does one converge and the other diverge?

Def

For $a < b < c$

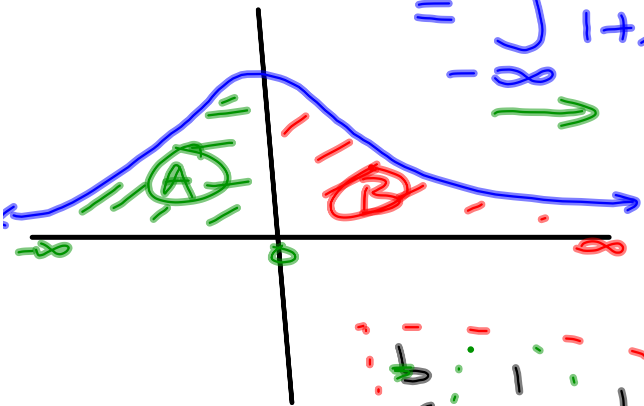
Given $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

If $\int_a^b f(x)\,dx$ or/and $\int_b^c f(x)\,dx$

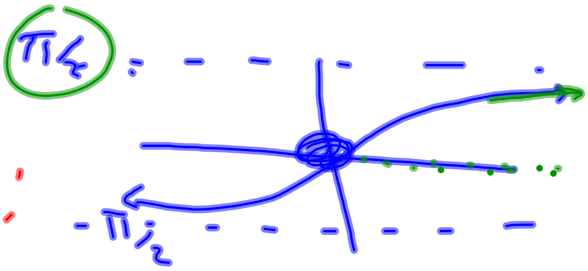
diverge then $\boxed{\int_a^c f(x)\,dx}$

Ex: $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$: convergent or divergent

$$= \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$



$$= 2\cdot \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$



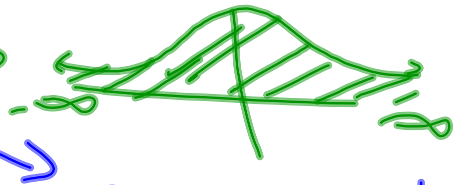
$$= 2\cdot \lim_{b\to\infty} \int_{0}^{b} \frac{1}{1+x^2}\,dx$$

$$= 2\cdot \lim_{b\to\infty} \Big[\operatorname{Arctan}(x)\Big]_{0}^{b}$$

$$= 2\cdot \lim_{b\to\infty} \Big[\operatorname{Arctan}(b) - \operatorname{Arctan}(0)\Big]$$

$$= 2\left[\frac{\pi}{2} - 0\right] = \boxed{\pi}$$

$$= \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx \quad \boxed{=\pi}$$



Convergent!

[STOP]