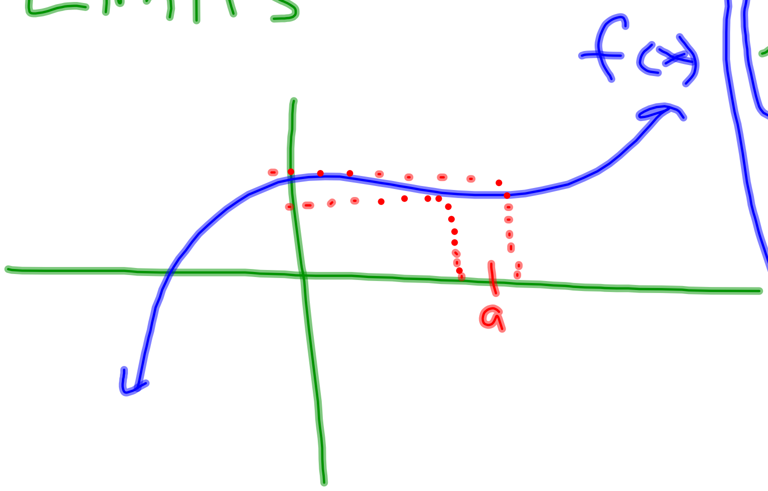
$\boxed{10.1}$ Limits and L'Hospital's Rule

Limits



$$\boxed{\lim_{x \to a} f(x)} =$$

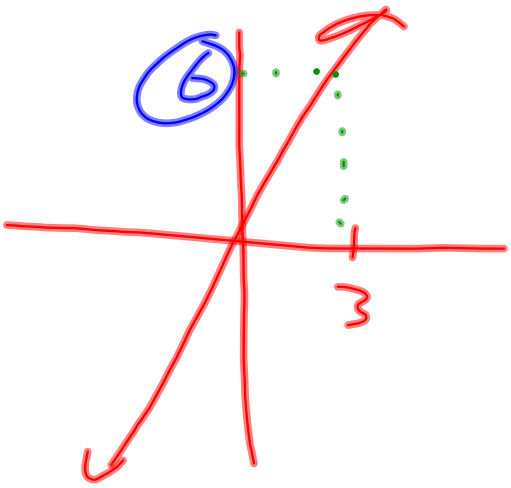
what does $f(x)$ get close to when x gets close to a

Examples

direct sub

(a)

$$\lim_{x \to 3} 2x = 2(3) = 6$$



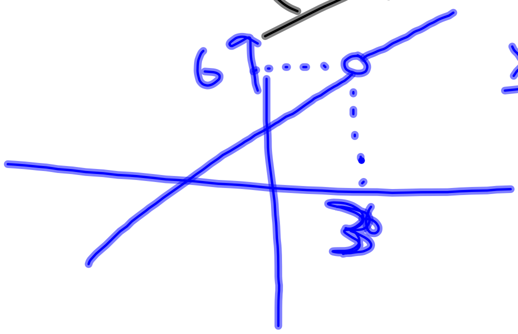
$$\lim_{x \to 3} 2x = 6$$

(b)

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \cancel{\frac{0}{0}}$$

Factor

$$\lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = \lim_{x\to 3}(x+3)$$



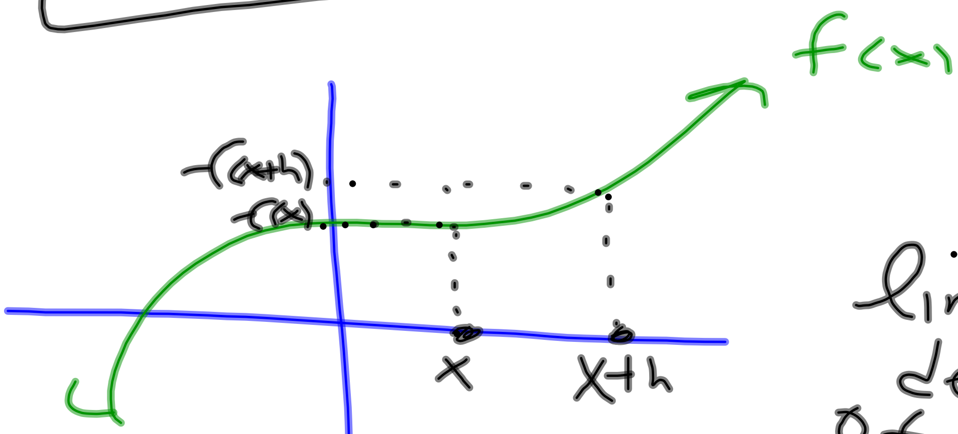
$\frac{x^2-9}{x-3}$

$= (3+3)$

$= 6$

(c)

$$\boxed{\lim_{h\to 0} \frac{f(x+h)-f(x)}{(x+h)-x}}$$



limit definition of derivative

$$\boxed{= f'(x)}$$

(d)

$$\lim_{x \to 0} \frac{\sin(x)}{x}$$

direct sub? $\frac{\sin(0)}{0} = \frac{0}{0}$ X

factoring? CANNOT!!

$\boxed{\text{L'Hospital's Rule}}$

L'Hospital's Rule

If: $\boxed{\checkmark}$ $\lim_{x \to a} f(x) = 0$

$\boxed{\checkmark}$ $\lim_{x \to a} g(x) = 0$

$\boxed{\checkmark}$ $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

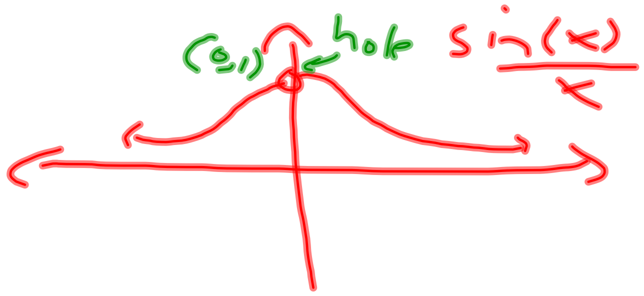
$$\lim_{x\to 0} \frac{\sin(x)}{x}$$

$$= \lim_{x\to 0} \frac{\cos(x)}{1}$$

$$= \frac{\cos(0)}{1}$$

$$\boxed{= 1}$$

- ☑ $\lim_{x\to 0} \sin(x) = 0$
- ☑ $\lim_{x\to 0} x = 0$
- ☑ $\lim_{x\to 0} \frac{\cos(x)}{1}$ exists



Using L'Hospital

Find $\lim_{x \to 1} \dfrac{x^3 - 3x + 2}{1 - x + \ln x}$

$= \lim_{x \to 1} \dfrac{3x^2 - 3}{-1 + 1/x}$

$= \lim_{x \to 1} \dfrac{6x}{-1 \cdot x^{-2}}$

$= \dfrac{6}{-1} = -6$

Extending L'Hospital

if $\boxed{\lim_{x \to a} \frac{f(x)}{g(x)}} = \frac{0}{0}$ OR $\frac{\infty}{\infty}$

then $\lim_{x \to a} \frac{f(x)}{g(x)}$

$= \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Find

$$\lim_{x \to \infty} \frac{2x^2 - 3x + 1}{3x^2 + 5x - 2}$$

$$= \lim_{x \to \infty} \frac{4x - 3}{6x + 5}$$

$$= \lim_{x \to \infty} \frac{4}{6} = \frac{4}{6}$$

(b) Find $\boxed{\lim_{x\to\infty} x^2 e^{-x}}$

$$= \lim_{x\to\infty} \frac{x^2}{e^x}$$

$$= \lim_{x\to\infty} \frac{2x}{e^x}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{e^\infty} = \boxed{0}$$