STEPS TO TEST IF A POWER SERIES CONVERGES

$\left|\frac{a_{n+1}}{a_n}\right| < 1$

1) Use the Ratio Test to find the x-values for which the series converges absolutely. This is usually an open interval $(c - R, c + R)$ where R is the radius of convergence. In some instances, the series converges for all values of x. In some rare cases, the series only converges at $x = c$.

2) If the interval of convergence ~~is finite~~, test for convergence at each endpoint using one or more of the convergence tests we have developed in this section.

For what values of x do the following series converge?

a) $\sum_{n=1}^{\infty} nx^n$ b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$$a_n = nx^n$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$$

$$\frac{a_{n+1}}{a_n} = \boxed{\frac{(n+1)x^{n+1}}{n \cdot x^n}}$$

$$\lim_{n\to\infty} \frac{(n+1)\cdot x \cdot \cancel{x^n}}{n \cdot \cancel{x^n}} = \lim_{n\to\infty} \frac{nx + x}{n}$$

$$= \lim_{n\to\infty} \boxed{x} < 1$$

$$|x| < 1$$

$$-1 < x < 1$$

For what values of x do the following series converge?

a) $\sum_{n=1}^{\infty} nx^n$ b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$(-1, 1)$ → $x + 2x^2 + 3x^3 + 4x^4 + \ldots$

$x = -1$

$= (-1) + 2(-1)^2 + 3(-1)^3 + 4(-1)^4$

$\sum n(-1)^n$

$= -1 + 2 - 3 + 4 \ldots - 5 + 6 - 7$

$\lim_{n\to\infty} n(-1)^n \neq 0$ @ $x = -1$, Diverge

$\boxed{x = 1}$

$\sum_{n=1}^{\infty} n \cdot 1^n$

$\lim_{n\to\infty} n \cdot 1^n \neq 0$ Diverges

$\boxed{(-1, 1)}$

For what values of x do the following series converge?

a) $\sum_{n=1}^{\infty} nx^n$ b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$$\lim_{n\to\infty} \frac{\frac{x^{n+1}}{(n+1)^2}}{\frac{x^n}{n^2}} = \lim_{n\to\infty} \frac{x\cdot \cancel{x^n}}{(n+1)^2}\cdot\frac{n^2}{\cancel{x^n}}$$

$$= \lim_{n\to\infty} \frac{x\cdot n^2}{(n+1)^2}$$

$$= \lim_{n\to\infty} \frac{2xn}{2(n+1)}$$

$$= \lim_{n\to\infty} \frac{2x}{2} = \left|\boxed{x}\right| < 1$$

x: $(-1,1)$

For what values of x do the following series converge?

a) $\sum_{n=1}^{\infty} nx^n$ b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$x = -1$

$x = 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{1}{1} + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \dots$$

☑ $\lim_{n\to\infty} \frac{1}{n^2} = 0$ converges @ -1

☑ $a_{n+1} < a_n$

For what values of x do the following series converge?

a) $\sum_{n=1}^{\infty} nx^n$ b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$[-1, 1)$

?

$x = 1$

$$\sum_{n=1}^{\infty} \frac{(1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

p-series, $p = 2 > 1$

Converge!

Converge $[-1, 1]$