Most common Series: Geometric Series

$$\sum_{n=0}^{\infty} ar^n$$

n → variable

r → multiplier

a → constant → starting value!!

Find the values of S_n through n=4, given a=2.

$S_0 = 2 \cdot r^0 = 2$

$S_1 = S_0 + a_1 = 2 + 2 \cdot r^1 = 2(1+r^1)$

$S_2 = S_1 + a_2 = \boxed{2 + 2 \cdot r^1 + 2 \cdot r^2} = 2(1+r^1+r^2)$

$S_3 = S_2 + a_3 = 2 + 2 \cdot r^1 + 2 \cdot r^2 + 2 \cdot r^3$

$= 2(1+r^1+r^2+r^3)$

a

$S_4 = 2(1+r^1+r^2+r^3+r^4)$

Finding an expression for the sum of an infinite geometric series, f(n)=a*r^n.

$n\to\infty$

$$\sum_{n=0}^{n=\infty} a\cdot r^n$$

$$r(S_n = a + ar + ar^2 + ar^3 + \cdots + ar^n)r$$

$$rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n+1}$$

$a\cdot r^n \cdot r$

$$S_n - rS_n = a - ar^{n+1}$$

$$S_n = \frac{a(1-r^{n+1})}{1-r}$$

$$\frac{S_n(1-r)}{1-r} = \frac{a(1-r^{n+1})}{1-r}$$

$$\sum_{n=0}^{\infty} ar^n = \lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r}$$

$$\lim_{n\to\infty} [S_n] = \lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r}$$

$$\sum_{n=0}^{\infty} a\cdot r^n = \lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r}$$

Given the expression developed below, what happens if |r|>1?
What happens if it is |r|<1?

$f(n) = a \cdot r^n$

S_n geometric series

$$\sum_{n=0}^{\infty} ar^n = \lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r}$$

$|r|>1$, ex $r=2$

$$\lim_{n\to\infty}\left[\frac{a(1-2^{n+1})}{1-2}\right] = \infty$$

$S_n \to$ diverge

$|r|<1$, ex $r=-\frac{1}{2}$

$$\sum_{n=0}^{\infty} a\cdot\left(-\tfrac{1}{2}\right)^n = \lim_{n\to\infty} \frac{a\left(1-\left(-\frac{1}{2}\right)^{n+1}\right)}{1-\left(-\frac{1}{2}\right)}$$

S_n converge

$$= \frac{a(1-0)}{3/2}$$

$$\sum_{n=0}^{\infty} a \cdot r^n = \lim_{n\to\infty} \frac{a(1+r^{n+1})}{1-r}$$

If $|r|<1$, the geometric series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots$$

converges to $\dfrac{a}{1-r}$. If $|r| \geq 1$, the series diverges.

If $|r| < 1$

$$= \frac{a(1+0)}{1-r} = \frac{a}{1-r}$$

Is the following series convergent or divergent?

$$\boxed{\sum_{k=0}^{\infty} \frac{2}{(-3)^k}} \longrightarrow \boxed{\text{converge}}$$

$a \cdot r^n$

$$= \sum_{k=0}^{\infty} 2 \cdot \left(\frac{1}{-3}\right)^k$$

$r = -\frac{1}{3}$

$|r| = \left|-\frac{1}{3}\right| < 1$

$$S_n = \frac{a}{1-r} = \frac{2}{1+1/3} = \frac{2}{4/3} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

Convergent or divergent?

$$\boxed{\sum_{n=0}^{\infty} 1\left(\frac{3}{2}\right)^n}$$

$$a \cdot r^n$$

$$\sum_{n=0}^{\infty} 1\left(\frac{3}{2}\right)^n \qquad |r| = \left|\frac{3}{2}\right| > 1$$

diverge

In Exercises 1– 6, find the first five terms of the sequence of partial sums. Graph the sequence of partial sums. Does it appear that the series converges or diverges? If the series converges, find its sum.

4. $1-\frac{4}{3}+\frac{16}{9}-\frac{64}{27}+\frac{256}{81}-\cdots$

$$\sum_{n=0}^{\infty} a \cdot r^{n}$$

multiplier:

$\left(-\frac{4}{3}\right)$

$1\left(-\frac{4}{3}\right)=-\frac{4}{3}$

$\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)=\frac{16}{9}$

$\left(\frac{16}{9}\right)\left(-\frac{4}{3}\right)=-\frac{64}{27}$

$$\sum_{n=0}^{\infty} 1\cdot\left(-\frac{4}{3}\right)^{n}$$

$|r|=\left|-\frac{4}{3}\right|>1$

diverges!

In Exercises 7–10 a sequence $\{a_n\}$ is defined. Determine:

a) whether $\{a_n\}$ is convergent; b) whether $\sum_{n=1}^{\infty} a_n$ is convergent.

$\boxed{a_n \neq 0}$

9. $a_n = \dfrac{3n-1}{2n+1}$

$$\lim_{n\to\infty} \frac{3n-1}{2n+1}$$

$$\lim_{n\to\infty} \boxed{\frac{3}{2}}$$

a_n converges to $3/2$.

① $\sum_{n=1}^{\infty} a_n$ diverge

② S_n diverges

In Exercises 14 – 23, determine whether the given series converges or diverges. If the series converges, find the sum.

18. $\frac{1}{11} - \frac{10}{11^2} + \frac{100}{11^3} - \frac{1000}{11^4} + \cdots$

$$\sum_{n=0}^{\infty} ar^n \qquad a = \frac{1}{11} \qquad r = -\frac{10}{11}$$

$$= \frac{1}{11} - \frac{10}{11^2} + \frac{10^2}{11^3} - \frac{10^3}{11^4} + \cdots$$

$$= \frac{1}{11} - \frac{10}{11 \cdot 11} + \frac{10^2}{11 \cdot 11^2} + \frac{10^3}{11 \cdot 11^3}$$

$$\left(\frac{1}{11}\right)\left(-\frac{10}{11}\right) = \left(\frac{-10}{11^2}\right)\left(-\frac{10}{11}\right) = \frac{100}{11^3}$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{11}\right)\left(\frac{-10}{11}\right)^n \qquad |r| = \left|\frac{-10}{11}\right| < 1$$

Converges

$$\frac{a}{1-r} = \frac{\frac{1}{11}}{1 + \frac{10}{11}}$$

$$\frac{\frac{1}{11}}{\frac{21}{11}} = \frac{1}{21}$$

In Exercises 26–29, ~~a)~~ find the common ratio of the geometric series; b) write the function that gives the sum of the series; and ~~c) in a single viewing rectangle graph the partial sum S_3.~~

multiplier

27. $1 + 2x + 4x^2 + 8x^3 + \cdots$

$(2x)$

$\sum a \cdot r^n$

$a = 1$

$r = 2x$

$1(2x) = 2x$

$(2x)(2x) = 4x^2$

$(4x^2)(2x) = 8x^3$

$\frac{a_{n+1}}{a_n} = r$

$\frac{8x^3}{4x^2} = 2x$

$\frac{4x^2}{2x} = 2x$

✓ $\sum_{n=0}^{\infty} (1)(2x)^n$

Converge: $|r| < 1$

$|2x| < 1$

$\boxed{-\frac{1}{2} < x < \frac{1}{2}}$