What is a Sequence?

Any ordered list of numbers is called a **sequence.** For example, the following list of reciprocals

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$$ $\frac{1}{6}, \frac{1}{7}$

is a sequence called the **harmonic** sequence.

The domain is the set of positive integers. $\{1, 2, 3 \ldots\}$

We write $a_n=f(n)$ $a_n = f(n)$

The values of a sequence are called **TERMS** of a sequence

$a_4 = f(4) = 1/4$

The harmonic series above could be described by f(n)=

$f(n) = \frac{1}{n}$

And a_{15} would then equal...

$a_1 = \frac{1}{1}$

$a_2 = \frac{1}{2}$

$a_{15} = \frac{1}{15}$

Use limits to determine whether a sequence will converge or diverge.

Determine whether the following sequences converge or diverge.

a) $a_n = \frac{n}{n+1}$ b) $b_n = \frac{(-1)^n}{n}$ c) $c_n = (-1)^n$

converges

$\lim_{n\to\infty} a_n$

$\lim_{n\to\infty} \frac{n}{n+1}$

$= \lim_{n\to\infty} \frac{1}{1} = 1$

$\lim_{n\to\infty} \frac{(-1)^n}{n}$

$= \pm \lim_{n\to\infty} \frac{1}{n}$

$= \pm 0$

converges

n^{th} term

converge??

$\{(-1)^n\}$

$(-1)^1 = -1$

$(-1)^2 = 1$

$(-1)^3 = -1$

$(-1)^4 = 1$

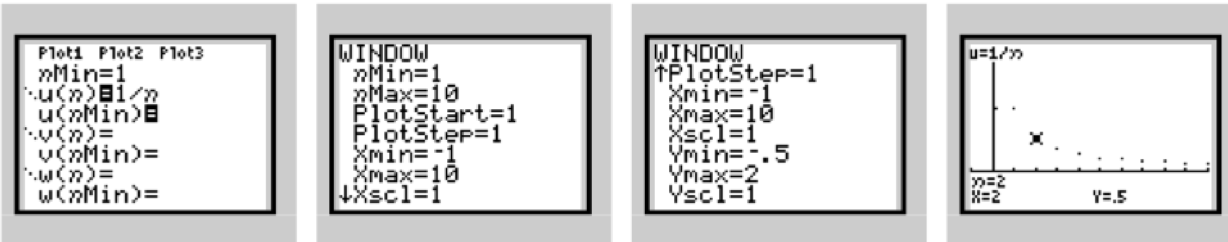
$\lim_{n\to\infty} (-1)^n$

$\hookrightarrow$ diverge

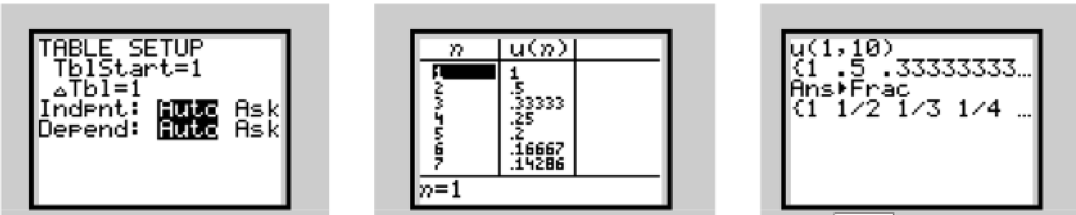
Evaluating and Graphing Sequences

Just as with functions, we can graphically and numerically investigate the behavior of a sequence. The TI-83 has a built-in sequence mode that makes defining and graphing sequences easier. We will use the harmonic sequence to illustrate these features.

From the Mode menu select **Seq** mode. Press **Y=** and enter u(*n*) = 1/*n*. Note that sequence functions must be defined in terms of the input variable *n* (use **X,T,θ,*n***). Set the window variables as shown, graph and trace. The horizontal axis is for the input variable *n* , while the vertical axis is for the terms of the sequence u(*n*).



Using the Table feature or from the Home screen, values for the harmonic sequence can be displayed



Press **2nd** [u] to obtain "u").

Example 2: *Determining Convergence and Divergencex*

Show graphically that the alternating harmonic sequence $a_n = \frac{(-1)^n}{n}$ converges to 0.

Series: The sum of all the sequences

$$S_n = \sum_{n=1}^{n=\infty} a_n = a_1 + a_2 + a_3 + \ldots$$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

We write the sum: S_n.

So, given $f(n)=n^2$, find S_3. 1st 3 term

$$S_3 = a_1 + a_2 + a_3$$

$$= (1)^2 + (2)^2 + (3)^2$$

$$\boxed{= 14}$$

Given: $f(n)=1/2^n$, find out if the series is convergent or divergent.

S_n: $\sum_{n=1}^{\infty} \frac{1}{2^n}$

Hint: look at S_1, S_2, S_3

as $n \uparrow$, what's happening S_n?

$S_1 = \frac{1}{2^1} = \boxed{\frac{1}{2}}$

$S_2 = \boxed{a_1 + a_2}$

$= \frac{1}{2} + \frac{1}{2^2}$

$= 3/4$

$S_3 = a_1 + a_2 + a_3$

$= 3/4 + \frac{1}{2^3}$

$= \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

$S_4 = S_3 + a_4$

$= \frac{7}{8} + \frac{1}{2^4}$

$= \frac{14}{16} + \frac{1}{16} = \frac{15}{16}$

$$\boxed{S_n = \frac{2^n - 1}{2^n}} \approx 1$$

$\frac{1}{n(n+1)}$

Given $f(n)=1/(n(n+1))$, find an expression for S_n. And then use that to determine whether S_n diverges or converges. If it converges, what does it converge to?

$$a_1 = S_1 = \frac{1}{1(1+1)} = \frac{1}{2}$$

$$a_2 = \frac{1}{2(2+1)} = \frac{1}{6}$$

$$S_2 = \frac{3}{6} + \frac{1}{6} = \frac{2}{3}$$

$$S_3 = S_2 + a_3$$
$$= \frac{2}{3} + \frac{1}{3(3+1)}$$
$$= \frac{8}{12} + \frac{1}{12} = 9/12 = 3/4$$

$$S_4 = S_3 + a_4$$
$$= 3/4 + \frac{1}{4(5)}$$
$$= \frac{15}{20} + \frac{1}{20} = \frac{16}{20} = 4/5$$

$$S_n = \frac{n}{n+1}$$

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)$$

$$\lim_{n\to\infty} \frac{1}{1} = 1$$

Converges

Simple test suggesting convergence. diverges

Consider that $S_n = S_{n-1} + a_n$. $S_4 = S_3 + a_4$

If S_n converges, what must be true about a_n as n gets huge??

if S_n converges, $\lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} a_n \neq 0$, then S_n diverges

If the series $\sum a_n$ converges, then $\lim_{n \to \infty} a_n = 0$.

The *n*th Term Test for Divergence

If $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum a_n$ diverges.

IMPORTANT: if $\lim_{n\text{->(infinity)}} a_n = 0$, the series does not necessarily converge.

Does the series below converge?
Hint: let 1/n=t

$\frac{1}{t} = n$

$$\sum_{n=1}^{\infty} n \sin\frac{1}{n}$$

$n = 1$

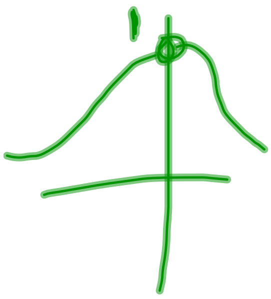
$t = \frac{1}{1} = 1$

$n = \infty;\ t = \frac{1}{\infty} = 0$

$$\sum_{t=1}^{t=0} \frac{1}{t} \sin(t)$$

a_n

$$\boxed{\lim_{n\to\infty} n \cdot \sin\left(\frac{1}{n}\right) = 1}$$



$$\lim_{t\to 0} \left(\frac{\sin(t)}{t}\right) = 1$$

$$\lim_{t\to 0} \frac{\cos(t)}{1} = 1$$

S_n diverges